NEMOTRON-CC-MATH: A 133 BILLION-TOKEN-SCALE HIGH QUALITY MATH PRETRAINING DATASET

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ABSTRACT

Pretraining large language models (LLMs) on high-quality, structured data such as mathematics and code substantially enhances reasoning capabilities. However, existing math-focused datasets built from Common Crawl suffer from degraded quality due to brittle extraction heuristics, lossy HTML-to-text conversion, and the failure to reliably preserve mathematical structure. In this work, we introduce Nemotron-CC-Math, a large-scale, high-quality mathematical corpus constructed from Common Crawl using a novel, domain-agnostic pipeline specifically designed for robust scientific text extraction.

Unlike previous efforts, our pipeline recovers math across various formats (e.g., MathJax, KaTeX, MathML) by leveraging layout-aware rendering with lynx and a targeted LLM-based cleaning stage. This approach preserves the structural integrity of equations and code blocks while removing boilerplate, standardizing notation into LaTeX representation, and correcting inconsistencies.

We collected a large, high-quality math corpus, namely Nemotron-CC-Math-3+ (133B tokens) and Nemotron-CC-Math-4+ (52B tokens). Notably, Nemotron-CC-Math-4+ not only surpasses all prior open math datasets—including MegaMath, FineMath, and OpenWebMath—but also contains 5.5× more tokens than FineMath-4+, which was previously the highest-quality math pretraining dataset. When used to pretrain a Nemotron-T 8B model, our corpus yields +4.8 to +12.6 gains on MATH and +4.6 to +14.3 gains on MBPP+ over strong baselines, while also improving general-domain performance on MMLU and MMLU-Stem.

We present the first pipeline to reliably extract scientific content—including math—from noisy web-scale data, yielding measurable gains in math, code, and general reasoning, and setting a new state of the art among open math pretraining corpora. To support open-source efforts, we release our code[1] and datasets [2].

1 INTRODUCTION

The rapid advancement of large language models (LLMs) has sparked a growing interest in improving their mathematical reasoning capabilities. Recent studies indicate that pretraining on carefully curated domain-specialized data—such as mathematics (Paster et al., 2024; Han et al., 2024; Wang et al., 2024b; Azerbayev et al., 2024) and code (Kocetkov et al., 2022; Lozhkov et al., 2024; Li et al., 2023) —substantially improves domain-specific accuracy, general knowledge and reasoning abilities (Muennighoff et al., 2023; Aryabumi et al., 2024; Lewkowycz et al., 2022; Shao et al., 2024). This suggests that high-quality mathematical data plays a pivotal role not only in solving math problems but also in strengthening broader reasoning skills.

Math capabilities of models like O1 (OpenAI, 2024) and DeepSeek-R1 (Guo et al., 2025) critically depend on access to large-scale, high-quality mathematical pretraining data. Unfortunately, datasets used in pretraining SOTA models like DeepSeekMath (Shao et al., 2024), Minerva (Lewkowycz

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[1]Code will be released as a part of Nemo-curator: https://github.com/NVIDIA-NeMo/Curator
[2]https://huggingface.co/datasets/nvidia/Nemotron-CC-Math-v1.

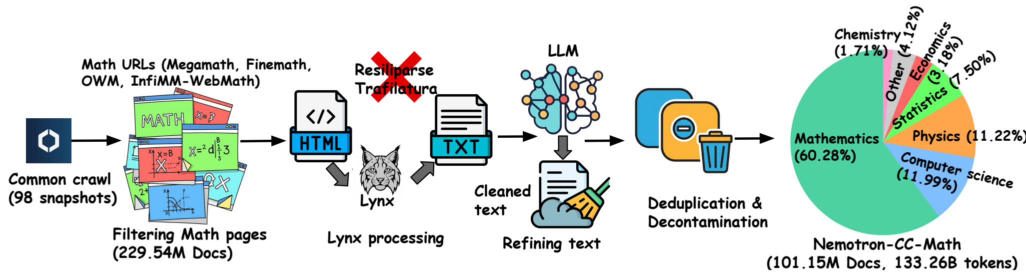

Figure 1: Overview of the Nemotron-CC-Math construction pipeline. Starting from Common Crawl snapshots, we extract math-related URLs using curated datasets (e.g., MegaMath, FineMath). After fetching 229.54M webpages, we render pages through `Lynx` and apply LLM-based cleaning, quality filtering, and deduplication (see §2.1). We visualize the topic distribution of our data (Right).

et al., 2022) and Qwen-2.5-Math (Yang et al., 2024) are not publicly released. Meanwhile, open-source alternatives such as OpenWebMath (OWM) (Paster et al., 2024), FineMath (Allal et al., 2025), InfiMMWebMath (Han et al., 2024) and MathPile (Wang et al., 2024b) remain limited in both scale and fidelity—largely due to brittle extraction pipelines that degrade content quality and fail to preserve mathematical equations and structure (see Appendix A.2).

While Common Crawl forms a primary source for large-scale pretraining (Penedo et al., 2023; Tang et al., 2024; Su et al., 2025), its value for mathematical pretraining remains underexploited. Existing math-specific extraction pipelines (Paster et al., 2024; Zhou et al., 2025) are not well-suited to fully leverage this resource. In particular, current methods frequently fail to detect or accurately extract equations, either omitting them altogether or corrupting their structure (Han et al., 2024; Allal et al., 2025). This severely compromises content fidelity. Mathematical notation on the web appears in a wide range of formats—including MathML, LaTeX, and dynamically rendered scripts—whose representations continue to evolve over time (see Figure 2). Compounding this challenge, HTML pages in Common Crawl often lack associated stylesheets and JavaScript resources, preventing proper rendering and further obstructing reliable equation recovery. These limitations collectively hinder the construction of high-quality mathematical pretraining corpora that capture the breadth and variety of real-world mathematical content.

To bridge this gap, we propose a modular, scalable, and domain-agnostic framework for reliably extracting mathematically rich content from raw web data, enabling the construction of a large-scale, diverse, and high-fidelity math corpus. Our multi-stage extraction and filtering pipeline ensures quality at scale (see Figure 1). In the first stage, HTML documents are rendered into text using the `Lynx` text-based browser[3], which preserves mathematical equations and symbols with high accuracy. In the second stage, a lightweight LLM normalizes heterogeneous math representations into LaTeX while discarding boilerplate and irrelevant content. This LLM-based approach allows us to avoid the brittle, heuristic-based rules employed in previous pipelines (Paster et al., 2024), resulting in more reliable and consistent extraction of mathematical content. Subsequently, we apply a quality classifier to retain the high-quality pages, followed by deduplication to eliminate redundancy. In addition, we perform thorough contamination detection against downstream benchmarks (see § 2.4), ensuring that any overlapping or duplicated samples are identified and removed from the corpus.

By leveraging the scale of Common Crawl and the rigor of our pipeline, we present Nemotron-CC-Math —the highest quality open-source math corpus to date, comprising of 133B tokens, where its highest quality subset (Nemotron-CC-Math-4+) totals 53B tokens. Our pipeline is optimized for performance using Polars and Ray, enabling us to process terabytes of HTML content efficiently. To facilitate future research, we release both the dataset and our full pipeline implementation.

Our contributions are as follows:

- We reviewed prior data extraction pipelines, and show that they fail to accurately extract math and code content, often stripping math equations and code snippets (Appendix A.2).
- We introduce a scalable and modular pipeline for extracting high-quality mathematical content from Common Crawl, explicitly addressing the longstanding challenge of HTML math variability—including LaTeX, MathML, Unicode, and inline or malformed equations.

[3] `https://lynx.invisible-island.net/`

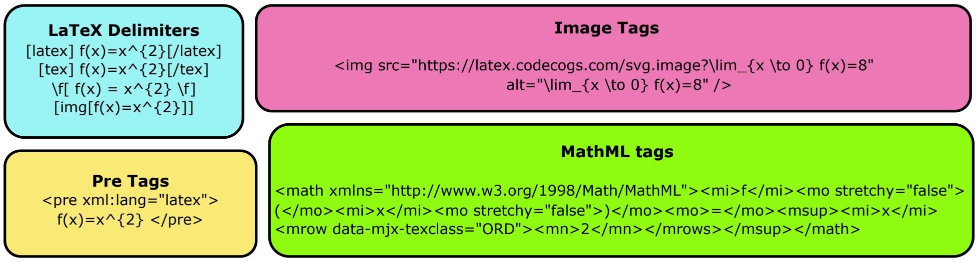

Figure 2: Mathematical expressions on HTML pages appear in diverse formats—LaTeX within custom delimiters, `<pre>` blocks, image tags, and MathML. These variations challenge standard text extraction pipelines, which often fail to recover the underlying LaTeX equations accurately.

- To our knowledge, this is the first work to employ the text-based browser `Lynx` for HTML-to-plain-text conversion with preservation of math and code formatting, and to introduce LLM-based standardization of mathematical representations across the content.
- We release Nemotron-CC-Math, a dataset of 133B tokens of high-quality and diverse math-rich web documents extracted from Common Crawl, whose 4+ subset contains 5.5× more tokens than FineMath-4+, the previous best math pretraining dataset.
- We open-source our full pipeline (extraction, processing and scoring) to ensure reproducibility and support its application to other domains.
- We thoroughly analyze Nemotron-CC-Math by examining its composition, including statistics on webpage types, subject areas, and the most frequent source domains.
- Through extensive experiments, we demonstrate that models pretrained on our dataset outperform those trained on existing pretraining math datasets across a range of benchmarks, including mathematics, code, and general knowledge tasks.

2 The Collection of Nemotron-CC-Math

We construct the Nemotron-CC-Math corpus from Common Crawl[4], a large-scale web archive extensively used in recent LLM training (Dubey et al., 2024; Hui et al., 2024; DeepSeek-AI et al., 2025). Common Crawl contains over 300B documents across more than 6M WARC files (each contains over 1GB of compressed content). Our goal is to build a pipeline that can process the technical content from Common Crawl correctly. We apply our pipeline to math domain to assemble a high-quality, large-scale corpus of mathematical content from Common Crawl. To achieve this, we designed a robust and highly scalable data processing pipeline capable of operating at the full scale of Common Crawl, as illustrated in Figure 1.

Prior efforts such as OWM (Paster et al., 2024) and DeepSeekMath (Shao et al., 2024) rely on lightweight classifiers to identify technical pages. We initially explored a similar approach but found fundamental limitations in achieving meaningful improvements: first, mathematical content constitutes $< 1\%$ of Common Crawl, making manual ground truth annotation extremely difficult; second, since classifiers must run on all Common Crawl documents, only very efficient methods like FastText with simplified HTML parsing are viable. This creates an inherently high-bias setup with no straightforward path to improvement—attempts to increase recall for technical content invariably lead to drastic drops in precision. Rather than refining such classifiers for marginal gains, we leverage community-filtered datasets: extracting URLs from OWM, InfiMM-WebMath (Han et al., 2024), FineMath (Allal et al., 2025), and MegaMath (Zhou et al., 2025), including all major subsets. This approach allows us to benefit from the diverse filtering strategies employed by different research groups while avoiding the limitations of any single classifier.

We then retrieve the original HTML from 98 Common Crawl snapshots (2014–2024) for these URLs, enabling fine-grained extraction that preserves mathematical expressions, symbols, and formatting—often degraded in prior processing (Appendix A.2). This process yields 229.54M high-quality webpages spanning a diverse range of mathematical content.

[4]`https://commoncrawl.org/`

2.1 Reliable Text Extraction For Scientific Content

2.1.1 Limitations of Prior Work

Extracting mathematical content from raw HTML presents a significant challenge for text extraction pipelines. Unlike natural language, which often follows consistent structural patterns, math equations appear in highly variable forms across the web (see Figure 2). These variations stem from the absence of standardized conventions for embedding math in HTML, as well as the diversity of rendering engines (e.g., MathJax, KaTeX, MathML, image-based representations, and custom plugins). Moreover, websites frequently evolve their rendering strategies, making any fixed set of heuristics fragile in practice. As a result, existing extraction pipelines often fail to reliably extract scientific content, with equations either missed entirely, mis-parsed, or distorted. Preserving formatting is equally important: the indentation and layout of code blocks and the placement of mathematical symbols often carry semantic meaning, and losing this structure severely degrades the value of the extracted content for downstream modeling.

Existing content extraction tools such as jusText (Endrédy & Novák, 2013), Trafilatura (Barbaresi, 2021), and Resiliparse (Bevendorff et al., 2018)—used in large-scale dataset construction pipelines including The Pile (Gao et al., 2020), FineMath (Allal et al., 2025), and RefinedWeb (Penedo et al., 2023)—were designed primarily for general-purpose boilerplate removal and narrative text extraction. While effective for general documents, they often strip or corrupt equations, miss inline LaTeX equations changing semantics, and flatten (or miss) code blocks requiring strict indentation (e.g., Python). These shortcomings limit their usability for building high-quality math or code datasets. Examples are provided in Appendix A.2.

2.1.2 Our Text Extraction Pipeline

The diversity of mathematical representations on the web necessitates using Large Language Models to faithfully convert technical HTML content into a format suitable for LLM pretraining. Since raw HTML from WARC files is too verbose for direct LLM processing, and traditional parsers risk losing critical information, we employ `lynx`, a text-based browser that renders web pages into plain text while preserving mathematical equations and code formatting. Unlike DOM-based parsers used in prior work (Paster et al., 2024; Zhou et al., 2025; Allal et al., 2025), `lynx` executes HTML layout rules to produce output that mirrors the human-perceived page structure, reliably capturing equations and maintaining code indentation.

While `lynx` preserves the structural layout of web pages, its output includes boilerplate elements such as navigation bars and redundant headers. To refine this output, we apply an LLM cleanup pass using Phi-4 (Abdin et al., 2024a)(14B parameters), which preserves primary content and references while removing non-essential content. LLM additionally standardizes mathematical expressions into consistent LaTeX format, and corrects typographical errors. This two-stage pipeline—structural preservation via `lynx` followed by semantic refinement via an LLM—yields high-quality, coherent text suitable for large-scale mathematical corpora. Ablation studies (§3.2) show that this cleanup task is simple enough for smaller models to perform effectively. Qualitative comparisons with prior work and the full cleanup prompt are provided in Appendices A.2 and A.3, respectively.

2.2 Quality Classification

To support the later stages of training where data fidelity is especially important (Hu et al., 2024; Abdin et al., 2024b), we further filtered our Nemotron-CC-Math to retain only its highest-scoring subset, Nemotron-CC-Math-4+. We employed the FineMath classifier (Allal et al., 2025) which assigns a 5-point scale score to each page, focusing on identifying content with mathematical reasoning and material suited to middle- and high-school levels. After classification, we also performed deduplicatication and decontamination (see §2.3 and §2.4). We developed two variants of Nemotron-CC-Math: Nemotron-CC-Math-4+ (52.32B tokens, 45M documents) with scores 4-5 and Nemotron-CC-Math-3+ (133.26B tokens, 101M documents) with scores 3-5.

2.3 Fuzzy Deduplication

Removing near-duplicate documents is essential for efficient and stable model training, and reducing the risk of memorization (Lee et al., 2022; Tokpanov et al., 2024). We applied fuzzy deduplication using the NeMo-Curator framework, which uses a MinHash-based Locality Sensitive Hashing

Table 1: Comparison of Nemotron-CC-Math with math pretraining datasets. Nemotron-CC-Math-4+ is 5.5× larger than the highest-quality open math dataset (FineMath-4+) with a permissive license, and substantially outperforms FineMath across math, code, and knowledge tasks (Table 2).

Dataset	**Open Source**	**#Documents (M)**	**#Tokens (B)**	**Source**
Minerva (Lewkowycz et al., 2022)	✗	-	38.50	arXiv, Web
MathMix (Lightman et al., 2023)	✗	-	1.50	Unknown
DeepSeekMath (Shao et al., 2024)	✗	-	120	CommonCrawl
ProofPile (Azerbayev et al., 2023)	✓	2.04	8.30	arXiv, Textbooks, Formal Math Libraries, StackExchange, ProofWiki, MATH
ProofPile-2 (Azerbayev et al., 2024)	✓	11.20	55	OpenWebMath, ArXiv, AlgebraicStack
AMPS (Hendrycks et al., 2021b)	✓	5.10	0.70	Khan Academy, Synthetic data
MathPile (Wang et al., 2024b)	✓	0.73	9.50	arXiv, Textbooks, ProofWiki, Wikipedia, StackExchange, CommonCrawl
OpenWebMath (Paster et al., 2024)	✓	6.30	14.70	CommonCrawl
InfiMM-WebMath-4+ (Han et al., 2024)	✓	6.30	8.50	CommonCrawl
FineMath-4+ (Allal et al., 2025)	✓	6.70	9.60	CommonCrawl
MegaMath-Pro (Zhou et al., 2025)	✓	15	15.10	CommonCrawl
Nemotron-CC-Math-4+ (Ours)	✓	45.10	52.32	CommonCrawl
InfiMM-WebMath-3+ (Han et al., 2024)	✓	13.90	20.50	CommonCrawl
FineMath-3+ (Allal et al., 2025)	✓	21.40	34	CommonCrawl
MegaMath-Web (Zhou et al., 2025)	✓	106.50	263.90	CommonCrawl
Nemotron-CC-Math-3+ (Ours)	✓	101.15	133.26	CommonCrawl

(LSH) (Broder, 2000) to efficiently detect duplicates. The probability that two documents with Jaccard similarity S hash to the same bucket is $P = 1 - (1 - S^b)^r$, where b is the number of hash functions per band and r is the number of bands. With r=20 bands and b=13 hash functions per band, our setup targets a Jaccard similarity threshold of 0.8. Pairwise similarity is computed using 24-character n-grams, and LSH uses concurrent shuffling of five bands to identify duplicates.

2.4 Decontamination

The source documents used in Nemotron-CC-Math are from mostly pre-decontaminated datasets. However, we follow a more thorough decontamination procedure as outlined in Yang et al. (2023). We embed all the documents in Nemotron-CC-Math using the Qwen2.5B 32B model (Qwen et al., 2025) as well as all the prompts and answers from our evaluation benchmarks: MMLU (Hendrycks et al., 2021a), MMLU-Pro (Wang et al., 2024a), MATH (Hendrycks et al., 2021b), and GSM8K (Cobbe et al., 2021). We remove all documents with a cosine similarity above 0.9 to any benchmark prompt or answer, resulting in the removal of less than 0.002% of all documents.

3 Experiments

Datasets We compare Nemotron-CC-Math to existing prior math pretraining datasets, including Megamath, OWM, and FineMath. Table 1 summarizes the dataset statistics.

Experimental Setup Math and code abilities generally arise only after extensive training; following Blakeney et al. (2024); Dubey et al. (2024); Ai2 (2024); Allal et al. (2025) to estimate the quality of different math pretraining datasets, we run annealing ablations on a mid training checkpoint of Nemotron-T 8B model (NVIDIA et al., 2025). The base model was pretrained on 9T tokens using a mixture of general-domain and math-focused corpora (see Appendix A.4 for a detailed breakdown). In each ablation, the target math dataset is upweighted to constitute 30% of the total data blend, while the weights of all other datasets are correspondingly downweighted to make up the remaining 70%. This controlled adjustment isolates the contribution of the math data while preserving overall blend composition (See Appendix A.5 for hyper-parameters). We consider two controlled ablations:

- **100B Token Ablation:** This setting targets compact, high-quality math datasets, typically below 30B tokens. For each run, the mathematical portion of the blend is replaced with a single candidate dataset—such as FineMath-4+, MegaMath-Pro, or OWM—enabling direct comparison with Nemotron-CC-Math-4+. The modified blends are trained for 100B tokens to evaluate performance under a consistent compute budget.
- **300B Token Ablation:** To fairly assess larger math datasets, including FineMath-3+ and MegaMath-Web, we apply the same replacement and proportional adjustment strategy but

extend the total annealing budget to 300B tokens. This configuration also tests whether increased scale can offset dataset quality differences.

Benchmarks We evaluate model performance across a diverse suite of benchmarks spanning knowledge understanding, code, and mathematical reasoning tasks. Knowledge understanding is assessed using MMLU datasets, including MMLU-Pro (Wang et al., 2024a), MMLU, and MMLU-STEM (Hendrycks et al., 2021a) with results reported as exact match (EM) accuracy. Code generation quality is measured on four tasks—MBPP (Austin et al., 2021), and HumanEval (Chen et al., 2021) and their EvalPlus variants, HumanEval+ and MBPP+ (Liu et al., 2023). For code tasks, following Guo et al. (2025), to improve the stability, we report the avg@20 which reports the average accuracy from generating 20 samples for each prompt. To produce these samples, we apply nucleus sampling with a temperature of 0.6 and a top-p value of 0.95. Mathematical reasoning is evaluated on the GMS8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021b) benchmarks, with greedy decoding and using Math-Verify[5] for symbolic matching. We run evaluations of all models ourselves using `lm-evaluation-harness`[6].

3.1 Pretraining Experiments Results

Results Tables 2 compare Nemotron-T 8B models pretrained with different math datasets at 100B and 300B tokens, respectively. Across all benchmarks, models using the curated Nemotron-CC-Math consistently match or outperform competing datasets—including OWM, MegaMath, and FineMath—on knowledge, code, and math tasks.

At 100B tokens, Nemotron-CC-Math-4+ achieves the top results in every math task—e.g., 40.6 on MATH (+4.8 vs. FineMath-4+, +6.6 vs. MegaMath-Pro) and 76.27 on GMS8K (+0.3 vs. FineMath-4+, +4.85 vs. OWM). It also leads most code benchmarks (e.g., 34.82 on HumanEval+, +2.3 vs. OWM) and all knowledge tasks (e.g., 38.49 on MMLU-Pro, +2.1 vs. MegaMath-Pro).

At 300B tokens, Nemotron-CC-Math-3+ extends these gains—reaching 44.2 on MATH (+9.6 vs. FineMath-3+, +12.6 vs. MegaMath-Web) and 80.06 on GMS8K (+0.6 vs. FineMath-3+, +3.6 vs. OWM). Code scores also improve substantially, with 37.16 on HumanEval+ (+3.0 vs. FineMath-3+) and 43.51 on MBPP+ (+4.6 vs. MegaMath-Web, +14.32 vs. Finemath-3+). Knowledge remains best or near-best across MMLU variants, with a top score of 64.26 on MMLU-STEM.

Although we do not explicitly target the code domain, it is noteworthy that the curated Nemotron-CC-Math datasets substantially improve code performance. Upon analysis, we find that Nemotron-CC-Math-3+ and Nemotron-CC-Math-4+ contain approximately 4.3M and 1.44M samples with code snippets[7]. In contrast to prior datasets, which often fail to capture code content, our curation pipeline retains code snippets in full, preserving syntax and structure. We attribute the observed code improvements to this incidental yet high-quality code data. Overall, results show that high-quality curated math data in pretraining boosts performance in math reasoning, code, and general knowledge. Comparing 100B and 300B token results, gains scale with more pretraining. This highlights the value of high-quality math data for improving LLMs across specialized and general domains.

3.2 Ablation On Model Choice

To ablate the model choice for the task of boilerplate removal from rendered web pages, we sampled 7M documents and evaluated several instruction-tuned LLMs including DeepSeek-V3 (Liu et al., 2024), Qwen2.5-32B/Instruct, Qwen2.5-72B/Instruct (Team, 2024), and Phi-4 across knowledge, coding, and math benchmarks.

Table 3 presents the results. Surprisingly, despite its significantly smaller size (14B parameters), Phi-4 performs competitively across all domains, often matching or exceeding the results of much larger models such as DeepSeek-V3 (671B) and Qwen2.5-72B-Instruct (72B). In particular, Phi-4 achieves the best performance on math tasks (e.g., 79.98 EM on GMS8K and 40.6 EM on MATH) and leads or matches the performance in several code-related benchmarks.

Given the marginal differences in performance and the substantial gap in computational cost, we selected Phi-4 as the default model for all experiments in this paper. Our findings indicate that

[5] `https://github.com/huggingface/math-verify`

[6] `https://github.com/EleutherAI/lm-evaluation-harness`.

[7] We filter out examples enclosed within triple backticks indicating a code block (e.g., “‘python ... “‘).

	Models Trained on 100B Tokens				
	Benchmark (Metric)	**OWM**	**MegaMath (Pro)**	**FineMath (4+)**	**Nemotron-CC-Math (4+)**
Knowledge	MMLU-Pro (EM)	35.49	36.41	36.74	**38.49**
	MMLU (EM)	65.62	66.81	66.73	**67.55**
	MMLU-Stem (EM)	58.83	60.86	61.62	**62.67**
Code	HumanEval+ (avg@20)	32.53	31.01	32.16	**34.82**
	MBPP+ (avg@20)	43.76	**46.03**	28.88	45.11
	MBPP (avg@20)	53.11	52.51	53.42	**53.48**
	HumanEval (avg@20)	37.07	35.91	37.77	**38.93**
Math	MATH (EM)	29.20	34.00	35.80	**40.60**
	GMS8K (EM)	71.42	73.46	75.97	**76.27**
	Models Trained on 300B Tokens				
	Benchmark (Metric)	**OWM**	**MegaMath (Web)**	**FineMath (3+)**	**Nemotron-CC-Math (3+)**
Knowledge	MMLU-Pro (EM)	35.00	36.33	**39.57**	39.32
	MMLU (EM)	65.20	65.44	67.92	**68.20**
	MMLU-Stem (EM)	59.20	59.88	62.29	**64.26**
Code	HumanEval+ (avg@20)	33.54	32.29	34.18	**37.16**
	MBPP+(avg@20)	37.59	38.89	29.19	**43.51**
	MBPP (avg@20)	52.22	53.05	**57.57**	56.15
	HumanEval(avg@20)	38.32	36.34	37.80	**40.30**
Math	MATH (EM)	34.20	31.60	34.60	**44.20**
	GMS8K (EM)	76.42	78.24	79.45	**80.06**

Table 2: Evaluation results for models trained with different math datasets using either 100B or 300B tokens. NEMOTRON-CC-MATH variants consistently outperform or obtain comparable results to OpenWebMath, MegaMath, and FineMath baselines across knowledge, code, and math tasks. Math performance improves with a longer token horizon, showing Nemotron-CC-Math continues to scale effectively with increased training. Code results use average accuracy over 20 generations; all other tasks use exact match (EM). Bold indicates the best result in each row.

the task of webpage boilerplate removal does not require excessively large models, and smaller instruction-tuned models can yield efficient and effective results.

3.3 DATASET ANALYSIS

Data Composition We measured domain distribution by document and character count. Table 4 shows the top twenty domains by each metric. Similar to prior work (Paster et al., 2024), the most common sources are discussion forums, Q&A sites, and educational resources. Overall, the dataset spans 980,922 unique domains, with the top 100 domains accounting for 36.46% of characters and 43.22% of documents.

	Benchmark (Metric)	**DeepSeek-V3**	**Qwen2.5-32B**	**Qwen2.5-72B**	**Phi-4**
Knowledge	MMLU-Pro (EM)	38.82	**39.65**	**39.65**	38.49
	MMLU (EM)	67.68	67.01	**67.73**	67.54
	MMLU-Stem (EM)	62.96	61.88	62.73	**63.24**
Code	HumanEval+ (avg@20)	28.54	28.35	29.63	**30.40**
	MBPP+ (avg@20)	**45.99**	38.58	41.34	41.88
	MBPP (avg@20)	53.91	53.83	54.09	**55.39**
	HumanEval (avg@20)	32.10	31.92	**35.21**	34.09
Math	MATH (EM)	36.60	38.80	38.60	**40.60**
	GMS8K (EM)	75.51	74.00	73.92	**79.98**

Table 3: Model choice ablation. We compare DeepSeek-V3 (671B), Qwen2.5-32B/72B, and Phi-4 (14B) across benchmarks. Despite its smaller size, Phi-4 performs competitively—often leading in math—demonstrating smaller models can efficiently clean webpages without losing performance.

Domain	#Documents (M)	Document %
mathhelpforum.com	8.54	8.44
jiskha.com	5.33	5.26
physicsforums.com	2.82	2.78
math.stackexchange.com	2.38	2.35
mathforum.org	2.38	2.35
openstudy.com	1.88	1.86
forums.wolfram.com	1.51	1.49
mathoverflow.net	1.33	1.32
nrich.maths.org	1.13	1.12
mathisfunforum.com	0.76	0.75
coursehero.com	0.76	0.75
brilliant.org	0.68	0.67
gmatclub.com	0.65	0.65
chegg.com	0.58	0.57
gradesaver.com	0.54	0.53
socratic.org	0.49	0.49
purplemath.com	0.45	0.44
physics.stackexchange.com	0.44	0.43
betterlesson.com	0.41	0.41
brainmass.com	0.40	0.40

Domain	#Characters (B)	Characters %
mathhelpforum.com	17.11	3.67
jiskha.com	12.52	2.69
mathforum.org	8.96	1.92
physicsforums.com	8.19	1.76
math.stackexchange.com	6.96	1.49
mathoverflow.net	6.78	1.45
nrich.maths.org	6.24	1.34
scribd.com	4.47	0.96
educator.com	3.58	0.77
forums.wolfram.com	3.35	0.72
docplayer.net	3.20	0.69
en.wikipedia.org	3.07	0.66
openstudy.com	3.10	0.66
gmatclub.com	2.73	0.59
mathisfunforum.com	2.72	0.58
coursehero.com	2.31	0.50
slideplayer.com	2.06	0.44
hindawi.com	1.98	0.42
softmath.com	1.95	0.42
archive.org	1.96	0.42

Table 4: Comparison of Most Common Domains by Document (left) and Character Count (right).

Topic Distribution To characterize the dataset, we randomly sampled 150,000 documents and classified them into Mathematics, Physics, Statistics, Chemistry, Economics, Computer Science, or Other using the Qwen3-30B-A3B-Instruct-2507 model (see Appendix A.1 for the prompt). Figure 1 shows the results. Most documents pertain to mathematics, with the remainder distributed across the other scientific domains; approximately 4.12% fall outside these categories.

4 Related Works

High-quality math pretraining datasets are essential for improving LLM reasoning. OWM compiles 14.7B tokens from Common Crawl but relies on brittle heuristics and Resiliparse for HTML rendering, often stripping or corrupting formulas and code. FineMath inherits these issues, building its 54B-token corpus using the OWM pipeline. Similarly, MegaMath faces similar challenges.

MathPile (Wang et al., 2024b) aggregates 9.5B tokens from sources including arXiv (85%), textbooks, and forums but much of the content remains in raw LaTeX form, limiting usability for LLM pretraining. InfiMM-Web-Math (Han et al., 2024) is 40B tokens multimodal dataset pairing images with math text. Proof-Pile (Azerbayev et al., 2023) is a 8.3B-token dataset collected from various sources such as arXiv, formal math libraries, Wikipedia and Stack Exchange. Proof-Pile-2 (Azerbayev et al., 2024) is a 55B-token dataset combining arXiv, OWM, and Algebraic-Stack mathematical code. Additionally, auxiliary datasets include AMPS (Hendrycks et al., 2021b) with Khan Academy problems and Mathematica-generated content, and NaturalProofs (Welleck et al., 2021), covering theorems and proofs from ProofWiki and the Stacks project.

Proprietary datasets like WebMath (OpenAI)(Polu & Sutskever, 2020), MathMix(Lightman et al., 2023), DeepSeekMath (Shao et al., 2024), and Minerva's Math Web Pages (Lewkowycz et al., 2022) advance math reasoning but lack public access, limiting transparency. We release Nemotron-CC-Math openly to foster community progress.

5 Conclusion

We present a scalable, domain-agnostic pipeline for extracting high-quality technical content from Common Crawl, focusing here on the mathematical domain. By integrating robust HTML-to-text conversion with LLM-based domain-aware cleaning, our approach addresses longstanding challenges in web-scale extraction of structured technical data.

Applied to the math domain, our pipeline produced Nemotron-CC-Math, whose 4+ subset is 5.5× larger than the previous highest-quality math set, FineMath-4+. Pretraining on Nemotron-CC-Math improves math reasoning (+12.6 MATH), code generation (+14.3 MBPP+), and general knowledge (+5.1 MMLU-Stem), outperforming prior math datasets.

Importantly, the modular, domain-agnostic design enables application to other technical fields. As LLMs advance, our pipeline offers a crucial tool for generating targeted, high-quality pretraining data to drive model capabilities.

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A APPENDIX

A.1 PROMPT USED FOR TOPIC CLASSIFICATION

We employ the following prompt to classify documents into a predefined set of categories. During classification, the large language model (LLM) occasionally produces category labels that fall outside the predefined taxonomy: Mathematics, Computer Science, Physics, Statistics, Economics, Chemistry, and Other. To maintain consistency and reduce label fragmentation, any out-of-taxonomy label is reassigned to the category Other, ensuring a coherent and structured category distribution.

```
You are a topic classification assistant.
Given the following document text, identify its main topic from this list only:
– Mathematics
– Computer Science
– Physics
– Statistics
– Chemistry
– Economics
– Other

Choose the single most relevant category from the list.
Document:
{text}

Your output should be only 1 word. Finish your response right after category and do not add any
    explanation.
```

A.2 QUALITATIVE EXAMPLES

This section presents qualitative comparisons among OpenWebMath(OWM), MegaMath-Pro, FineMath-4+, and Nemotron-CC-Math-4+, highlighting differences in content quality.

A.2.1 DEGENERATE CASES IN MEGAMATH-PRO DATASET

We identified a subset of degenerate generations within the MegaMath-Pro dataset. Representative examples are presented below to illustrate this phenomenon. Notably, these samples achieve unexpectedly high scores on both mathematical and language scores, raising concerns about the dataset's overall reliability for pretraining LLMs. For each example, we provide the associated metadata. The excerpts shown correspond to the initial portion of each generation; in every case, the text extends over several additional pages, repeating the final sentence displayed.

A.2.2 SIDE BY SIDE COMPARISON BETWEEN OUR DATASET AND PRIOR WORK

We observe that our pipeline not only keep the math equations but also keep the codes and their formatting. We observe that previous pipeline in most cases are not keeping codes or lose their formatting. This is specifically important for languages like python. To highlight this difference, we provide two sets of examples demonstrating both code and mathematical equations. Notably, inline equations are often removed in prior work, such as MegaMath.

A degenerate sample from MegaMath-Pro

The Integral Calculator lets you calculate integrals and antiderivatives of functions online — for free. It consists of more than 17000 lines of code. For a function f(x) of a real variable x, we have the integral Z b a f(x)dx. It signifies the area calculation to the x-axis from the curve. The Integral Calculator will show you a graphical version of your input while you type.
In ”Options”, you can set the variable of integration and the integration bounds. The Integral Calculator has to detect these cases and insert the multiplication sign. For each function to be graphed, the calculator creates a JavaScript function, which is then evaluated in small steps in order to draw the graph.
While graphing, singularities (e. g. poles) are detected and treated specially. The practice problem generator allows you to generate as many random exercises as you want. The Integral Calculator supports definite and indefinite integrals (antiderivatives) as well as integrating functions with many variables.
You can also check your answers using the integral calculator. The students should also familiar with line integrals. The calculator will simplify any complex expression, with steps shown. The Integral Calculator will show the result below.
The Integral Calculator is a simple online calculator that computes the definite and indefinite integrals. The Integral Calculator has to respect the order of operations. The Integral Calculator will show you a graphical version of your input while you type.
The Integral Calculator is a free online tool for calculating the value of a definite integral. The Integral Calculator, part of the graphing calculator, helps with one variable calculus. The Integral Calculator supports definite and indefinite integrals (antiderivatives) as well as integrating functions with many variables.
The Integral Calculator is able to calculate integrals online of the composition of common functions, using integral properties, the different mechanisms of integration and calculation online. The Integral Calculator is a simple online calculator that computes the definite and indefinite integrals. The Integral Calculator will show you a graphical version of your input while you type.
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The Integral Calculator is a free online tool for calculating the value of a definite integral.

Meta:

URL: `http://031c82c.netsolhost.com/yvcmr4/article.php?c08ee4=complex-integration-calculator`

Math Score: 0.9996713399887085

Lang Score: 0.7893766164779663

WARC Filename: CC-MAIN-2022-21/segments/1652662531762.30/warc/CC-MAIN-20220520061824-00605.warc.gz

A degenerate sample from MegaMath-Pro

About $113, Starett. Here is the online trigonometry cosecant calculator to find the cosecant of an angle. Another way to calculate the exterior angle of a triangle is to subtract the angle of the vertex of interest from 180°. The angle will be calculated and displayed. In general, try to use the law of sines first. The reference angle must be 90°. Use the law of cosines to find the missing measurements of the triangles in these two examples.

To find the angle set by your sliding bevel, the best tool to use is a standard protractor. Begin by measuring the angle of the driveshaft and writing it down. The law of cosines can be used to find the measure of an angle or a side of a non-right triangle if we know: two sides and the angle between them or three sides and no angles.

The law of sines is an important relationship between the angle measures and side lengths of non-right triangles. The law of cosines can be used to find an unknown angle or an unknown side of a triangle. The given angle may be in degrees or radians. The reference angle must be 90°.

The angle will be set and you will now simply enter the length of the second Wall. You will see the following Angle Calculator routine appear. Use of calculator to find the Quadrant of an angle 1 - enter the angle: in degrees top input. If you enter a quadrantal angle, the result is the angle in the first Quadrant with reference to the x-axis.

A: By taking three measurements for the adjoining Walls, Solid can calculate the angle for you.

Q: For a non-standard layout, how can I place two adjoining Walls without knowing the angle?

The result for ”angle 1” will be the stair angle.

The angle will be calculated and displayed. Use the outer arc if the angle you’re measuring opens to the left. Use the inner arc if the angle you’re measuring opens to the right. The straight edge is where the pivot point begins and where you will compute and determine angles.

The angle will be set and you will now simply enter the length of the second Wall. You will see the following Angle Calculator routine appear. Now select OK. The angle will be calculated and displayed.

To determine the number of degrees in the angle, simply count the number of degrees between the two sides. The angle will be calculated and displayed. Use the law of cosines to find one of the angles.

The angle will be calculated and displayed. Use the law of cosines to find the missing measurements of the triangles in these two examples. The angle will be calculated and displayed. The angle will be calculated and displayed.

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Meta:

URL: `http://102theking.com/regina-ward-sbolp/how-to-find-an-angle-without-an-angle-finder-ebb037`

Math Score: 0.9885214567184448

Lang Score: 0.8642399311065674

WARC Filename: CC-MAIN-2021-31/segments/1627046154796.71/warc/CC-MAIN-20210804045226-00255.warc.gz

A degenerate sample from MegaMath-Pro

The axis of symmetry in a vertical parabola is a vertical line. The equation of the axis of symmetry in a vertical parabola is equal to the x-coordinate of the vertex. Every parabola has an axis of symmetry which is the line that divides the graph into two perfect halves. The axis of symmetry always passes through the vertex of the parabola. The x-coordinate of the vertex is equal to the formula.

To learn about the axis of symmetry, watch this tutorial! The axis of symmetry is the line that divides the graph into two perfect halves. The axis of symmetry is always a vertical line of the form x = n, where n is a real number.

A parabola is the graph of a quadratic function. Each parabola has a line of symmetry. Also known as the axis of symmetry, this line divides the parabola into mirror images. The line of symmetry is always a vertical line of the form x = n, where n is a real number.

When graphing, we want to include certain special points in the graph. The y-intercept is the point where the graph intersects the y-axis. The x-intercepts are the points where the graph intersects the x-axis. The vertex is the point that defines the minimum or maximum of the graph.

The axis of symmetry for an equation with x 2 is the vertical line that passes through the vertex. The axis of symmetry is the line x = h, where (h, k) is the vertex of the parabola.

The axis of symmetry is the line that divides the graph into two perfect halves. The axis of symmetry is always a vertical line of the form x = n, where n is a real number.

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Meta:

URL: `http://1798091312.srv040122.webreus.net/q8oh94/c5d31c-vertical-symmetry-graph`

Math Score: 0.9988757371902466

Lang Score: 0.9151933789253235

WARC Filename: CC-MAIN-2021-25/segments/1623488273983.63/warc/CC-MAIN-20210621120456-00277.warc.gz

Lynx output

#10000 Terabyte

10000 Terabyte

about opensource disclaimer

(BUTTON)
about opensource disclaimer

Detailed explanation of a smart solution to an algo problem beating 99.9% submission

Written on January 7th, 2018 by @10000TB
[attachments_article_algorithm_col_slide_lamparas−colgantes−algorithm−slide−03.jpg]

This post is about a coding problem and why the solution I pasted down below is smart.
Problem:
Given two sparse matrices A and B, return the result of AB.

You may assume that A's column number is equal to B's row number.

Example:

```
A = [
  [ 1, 0, 0],
  [-1, 0, 3]
]

B = [
  [ 7, 0, 0 ],
  [ 0, 0, 0 ],
  [ 0, 0, 1 ]
]


     |  1 0 0 |   | 7 0 0 |   |  7 0 0 |
AB = | -1 0 3 | x | 0 0 0 | = | -7 0 3 |
                  | 0 0 1 |
```

If it is of your interest, I would recommend you take a few minutes to think about how you would approach this problem!

The main focus of this post is to 1)explain in detail why the provided solution is smart and 2)make some improvements/tweaks in the code of the smart solution to show you which part is really essential, 3) also i will briefly mention why Sparse Matrix Manipulation can help make some improvements on top of the smart solution.

a) Originally, the normal way to calculate the multiplication of two metrics A, and B is as follow: We take the the all values from the first line of A, and all values from the first column of B, and multiply the corresponding values and sum them up, the final sum is the value for the location of first column and first row in final result matrix. Similarly, the value at [i][j] of result matrix C, which is C[i][j] is calculated as:

C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + ... A[i][K]B[K][j]

(which is the sum of each multiplication of corresponding K values from row i of A and K values from column j of B)

The Key is: if we calculate it this way, we finishing calculating the final value for the result matrix at once

Then a brute force solution is as follow:

```
public class Solution {
    public int[][] multiply(int[][] A, int[][] B) {
        int m = A.length, n = A[0].length, nB = B[0].length;
        int[][] C = new int[m][nB];

        for(int i = 0; i < m; i++) {
            for (int j = 0; j < nB; j++) {
                for(int k = 0; k < n; k++) {
                    C[i][j] += A[i][k] * B[k][j];
                }
            }
        }
        return C;
    }
}
```

b) The smart solution: the key part of smart solution is that: it does not calculate the final result at once, and it takes each value from A, and calculate and partial sum and accumulate it into the final spot:

For example, for each value A[i][k], if it is not zero, it will be used at most nB times (n is B[0]. length), which can be illustrated as follow: Generally for the following equation:

C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + ... A[i][k]B[k][j] .... A[i][K]B[K][j]

j can be from 0 to nB, if we write all of them down, it will like following:

For i from 0 to nB:

C[i][0]=A[i][0]B[0][0] + A[i][1]B[1][0] + A[i][2]B[2][0] + ... A[i][k]B[k][0] .... A[i][K]B[K][0]

C[i][1]=A[i][0]B[0][1] + A[i][1]B[1][1] + A[i][2]B[2][1] + ... A[i][k]B[k][0] .... A[i][K]B[K][1]

...

C[i][nB]=A[i][0]B[0][nB] + A[i][1]B[1][nB] + A[i][2]B[2][nB] + ... A[i][k]B[k][nB] .... A[i][K]*B[K][nB]

As you can see from above: for the same value A[i][k] from the first matrix, it will be used at most nB times if A[i][k] is not zero. And the smart solution is taking advantage of that!!!, the smart solution can be described as:

For each value A[i][k] in matrix A, if it is not zero, we calculate A[i][k] * B[k][j] and accumulate it into C[i][j] (Key part: the C[i][j] by now is not the final value in the result matrix !! Remember, in the brute force solution, the final value of C[i][j], takes sum of all multiplication values of K corresponding values from A and B? here C[i][j] is only sum of some multiplication values, NOT ALL until the program is done)

BY NOW, it is very clear that, if the value A[i][k] from matrix is zero, we skip a For−loop− calculation, which is a loop iterating nB times, and this is the key part of why the smart solution is smart!!!

The smart solution code is as follow:

```
public class Solution {
  public int[][] multiply(int[][] A, int[][] B) {
    int m = A.length, n = A[0].length, nB = B[0].length;
    int[][] C = new int[m][nB];

    for(int i = 0; i < m; i++) {
      for(int k = 0; k < n; k++) {
        if (A[i][k] != 0) {
          for (int j = 0; j < nB; j++) {
            if (B[k][j] != 0) C[i][j] += A[i][k] * B[k][j];
          }
        }
      }
    }
    return C;
  }
}
```

(Credit:@yavinci; I am having a different version of the solution, so I am directly referencing the original version as a reference to demonstrate how mine is different).

Based on the discussion above, the inner checking (if (B[k][j] != 0)) is actually not necessary, because whether or not we have that check, we still iterate nB times, (since the operation C[i][j] += A[i][k] * B[k][j]; inside the if−check is O(1) time)

So the smart solution can also be written as follow by removing the check (which is my version):

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public class Solution {
    public int[][] multiply(int[][] A, int[][] B) {
        int m = A.length, n = A[0].length, nB = B[0].length;
        int[][] C = new int[m][nB];

        for(int i = 0; i < m; i++) {
            for(int k = 0; k < n; k++) {
                if (A[i][k] != 0) {
                    for (int j = 0; j < nB; j++) {
                        if (B[k][j] != 0) C[i][j] += A[i][k] * B[k][j];
                    }
                }
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c) "Sparse matrix manipultion" helps, if we compress the first sparse matrix into rows of lists(in each row list, it contains (value, index) pair), we actually don't need to go over all values in a row in matrix A when are calculating the final result matrix. But Overall, it does not help improve run time algorithmatically!!

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A sample from Nemotron-CC-Math

10000 Terabyte

Detailed Explanation of a Smart Solution to an Algorithm Problem

Written on January 7th, 2018 by @10000TB

This post is about a coding problem and why the solution provided is smart. The problem is as follows:

Problem: Given two sparse matrices \(A \) and \(B \), return the result of \(AB \). You may assume that \(A \)'s column number is equal to \(B \)'s row number.

Example:

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
−1 & 0 & 3
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
7 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
AB = \begin{bmatrix}
1 & 0 & 0 \\
−1 & 0 & 3
\end{bmatrix}
\times
\begin{bmatrix}
7 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
=
\begin{bmatrix}
7 & 0 & 0 \\
−7 & 0 & 3
\end{bmatrix}
\]

If you are interested, consider how you would approach this problem!

The main focus of this post is to:
1. Explain in detail why the provided solution is smart.
2. Make some improvements/tweaks in the code of the smart solution to show which part is really essential.
3. Briefly mention why Sparse Matrix Manipulation can help make improvements.

a) Brute Force Solution

The normal way to calculate the multiplication of two matrices \(A \) and \(B \) is as follows: We take all values from the first row of \(A \) and all values from the first column of \(B \), multiply the corresponding values and sum them up. The final sum is the value for the location of the first row and first column in the final result matrix \(C \). Similarly, the value at \(C[i][j] \) is calculated as:

\[
C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + \ldots + A[i][K]B[K][j]
\]

The brute force solution is as follows:

```java
public class Solution {
    public int[][] multiply(int[][] A, int[][] B) {
        int m = A.length, n = A[0].length, nB = B[0].length;
        int[][] C = new int[m][nB];

        for (int i = 0; i < m; i++) {
            for (int j = 0; j < nB; j++) {
                for (int k = 0; k < n; k++) {
                    C[i][j] += A[i][k] * B[k][j];
                }
            }
        }
        return C;
    }
}
```

b) The Smart Solution

The key part of the smart solution is that it does not calculate the final result at once. Instead, it takes each value from \(A \), calculates a partial sum, and accumulates it into the final spot.

For example, for each value \(A[i][k] \), if it is not zero, it will be used at most \(nB \) times (\(n \) is \(B[0].length \)). Generally, for the following equation:

\[
C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + \ldots + A[i][k]B[k][j] + \ldots + A[i][K]B[K][j]
\]

\(j \) can be from 0 to \(nB \). If we write all of them down, it will look like this:

For \(i \) from 0 to \(nB \):

\[
C[i][0] = A[i][0]B[0][0] + A[i][1]B[1][0] + A[i][2]B[2][0] + \ldots + A[i][k]B[k][0] + \ldots + A[i][K]B[K][0]
\]

\[
C[i][1] = A[i][0]B[0][1] + A[i][1]B[1][1] + A[i][2]B[2][1] + \ldots + A[i][k]B[k][1] + \ldots + A[i][K]B[K][1]
\]

\[
\ldots
\]

\[
C[i][nB] = A[i][0]B[0][nB] + A[i][1]B[1][nB] + A[i][2]B[2][nB] + \ldots + A[i][k]B[k][nB] + \ldots + A[i][K]B[K][nB]
\]

As you can see, for the same value \(A[i][k] \) from the first matrix, it will be used at most \(nB \) times if \(A[i][k] \) is not zero. The smart solution takes advantage of this by calculating \(A[i][k] \times B[k][j] \) and accumulating it into \(C[i][j] \). Note that \(C[i][j] \) is not the final value in the result matrix at this point. In the brute force solution, the final value of \(C[i][j] \) is the sum of all multiplication values of corresponding values from \(A \) and \(B \); here, \(C[i][j] \) is only the sum of some multiplication values, not all, until the program is done.

The smart solution code is as follows:

```java
public class Solution {
    public int[][] multiply(int[][] A, int[][] B) {
        int m = A.length, n = A[0].length, nB = B[0].length;
        int[][] C = new int[m][nB];

        for (int i = 0; i < m; i++) {
            for (int k = 0; k < n; k++) {
                if (A[i][k] != 0) {
                    for (int j = 0; j < nB; j++) {
                        if (B[k][j] != 0) C[i][j] += A[i][k] * B[k][j];
                    }
                }
            }
        }
        return C;
    }
}
```

(Credit: @yavinci; I am having a different version of the solution, so I am directly referencing the original version as a reference to demonstrate how mine is different.)

Based on the discussion above, the inner checking \((if (B[k][j] \neq 0)) \) is actually not necessary because whether or not we have that check, we still iterate \(nB \) times. The operation \(C[i][j] += A[i][k] \times B[k][j] \) inside the if−check is \(O(1) \) time. So the smart solution can also be written as follows by removing the check (which is my version):

```java
public class Solution {
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        int[][] C = new int[m][nB];

        for (int i = 0; i < m; i++) {
            for (int k = 0; k < n; k++) {
                if (A[i][k] != 0) {
                    for (int j = 0; j < nB; j++) {
                        C[i][j] += A[i][k] * B[k][j];
                    }
                }
            }
        }
        return C;
    }
}
```

c) Sparse Matrix Manipulation

Sparse matrix manipulation helps if we compress the first sparse matrix into rows of lists (in each row list, it contains (value, index) pairs). We actually don’t need to go over all values in a row in matrix \(A \) when calculating the final result matrix. However, overall, it does not help improve runtime algorithmically.

References

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Problem:

Given two sparse matrices A and B, return the result of AB.

You may assume that A's column number is equal to B's row number.

Example:

A = [
[1, 0, 0],
[−1, 0, 3]
]

B = [
[7, 0, 0],
[0, 0, 0],
[0, 0, 1]
]

| 1 0 0 | | 7 0 0 | | 7 0 0 |
AB = | −1 0 3 | x | 0 0 0 | = | −7 0 3 |
| 0 0 1 |

If it is of your interest, I would recommend you take a few minutes to think about how you would approach this problem!

The main focus of this post is to 1)explain in detail why the provided solution is smart and 2)make some improvements/tweaks in the code of the smart solution to show you which part is really essential, 3) also i will briefly mention why Sparse Matrix Manipulation can help make some improvements on top of the smart solution.

a) Originally, the normal way to calculate the multiplication of two metrics A, and B is as follow: We take the the all values from the first line of A, and all values from the first column of B, and multiply the corresponding values and sum them up, the final sum is the value for the location of first column and first row in final result matrix. Similarly, the value at [i][j] of result matrix C, which is C[i][j] is calculated as:

C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + ... A[i][K]B[K][j]
(which is the sum of each multiplication of corresponding K values from row i of A and K values from column j of B)
The Key is: if we calculate it this way, we finishing calculating the final value for the result matrix at once

Then a brute force solution is as follow:

b) The smart solution: the key part of smart solution is that: it does not calculate the final result at once, and it takes each value from A, and calculate and partial sum and accumulate it into the final spot:
For example, for each value A[i][k], if it is not zero, it will be used at most nB times (n is B[0].length), which can be illustrated as follow: Generally for the following equation:

C[i][j] = A[i][0]B[0][j] + A[i][1]B[1][j] + A[i][2]B[2][j] + ... A[i][k]B[k][j] .... A[i][K]B[K][j]

j can be from 0 to nB, if we write all of them down, it will like following:

For i from 0 to nB:

C[i][0]=A[i][0]
B[0][0] + A[i][1]B[1][0] + A[i][2]B[2][0] + ... A[i][k]B[k][0] .... A[i][K]B[K][0]
C[i][1]=A[i][0]
B[0][1] + A[i][1]B[1][1] + A[i][2]B[2][1] + ... A[i][k]B[k][0] .... A[i][K]B[K][1]

C[i][nB]=A[i][0]
B[0][nB] + A[i][1]B[1][nB] + A[i][2]B[2][nB] + ... A[i][k]B[k][nB] .... A[i][K]*B[K][nB]

As you can see from above: for the same value A[i][k] from the first matrix, it will be used at most nB times if A[i][k] is not zero. And the smart solution is taking advantage of that!!!, the smart solution can be described as:

For each value A[i][k] in matrix A, if it is not zero, we calculate A[i][k] * B[k][j] and accumulate it into C[i][j] (Key part: the C[i][j] by now is not the final value in the result matrix !! Remember, in the brute force solution, the final value of C[i][j], takes sum of all multiplication values of K corresponding values from A and B? here C[i][j] is only sum of some multiplication values, NOT ALL until the program is done)

BY NOW, it is very clear that, if the value A[i][k] from matrix is zero, we skip a For−loop−calculation, which is a loop iterating nB times, and this is the key part of why the smart solution is smart!!!

The smart solution code is as follow:

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Based on the discussion above, the inner checking (if (B[k][j] != 0)) is actually not necessary, because whether or not we have that check, we still iterate nB times, (since the operation C[i][j] += A[i][k] * B[k][j]; inside the if−check is O(1) time)

So the smart solution can also be written as follow by removing the check (which is my version):

c) "Sparse matrix manipultion" helps, if we compress the first sparse matrix into rows of lists(in each row list, it contains (value, index) pair), we actually don't need to go over all values in a row in matrix A when are calculating the final result matrix. But Overall, it does not help improve run time algorithmatically!!

References:

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A sample from MegaMath-Pro

Detailed Explanation of a Smart Solution to a Matrix Multiplication Problem

The problem involves multiplying two sparse matrices A and B. We are given that A's column number is equal to B's row number.

Problem Statement

Given two sparse matrices A and B, return the result of AB.

Example:
A = [
[1, 0, 0],
[−1, 0, 3]
]
B = [
[7, 0, 0],
[0, 0, 0],
[0, 0, 1]
]
The result AB is calculated as follows:
AB = [
[7, 0, 0],
[−7, 0, 3]
]

Normal Approach to Matrix Multiplication

The normal way to calculate the multiplication of two matrices A and B is to take the values from the first line of A and the first column of B, multiply the corresponding values, and sum them up. The final sum is the value for the location of the first column and first row in the final result matrix.

C[i][j] = A[i][0]∗B[0][j] + A[i][1]∗B[1][j] + A[i][2]∗B[2][j] + ... + A[i][K]∗B[K][j]

Brute Force Solution

The brute force solution involves calculating the final result at once.

Smart Solution

The smart solution does not calculate the final result at once. Instead, it takes each value from A, calculates a partial sum, and accumulates it into the final spot.

For each value A[i][k], if it is not zero, it will be used at most nB times (where n is B[0].length). The smart solution can be described as follows:

For each value A[i][k] in matrix A, if it is not zero, calculate A[i][k] ∗ B[k][j] and accumulate it into C[i][j].

The key part of the smart solution is that it skips a For−loop calculation if the value A[i][k] from matrix A is zero. This is the key part of why the smart solution is smart.

Smart Solution Code

The smart solution code can be written as follows:

Improvement Using Sparse Matrix Manipulation

Sparse matrix manipulation can help improve the solution by compressing the first sparse matrix into rows of lists (in each row list, it contains a (value, index) pair). This allows us to avoid going over all values in a row in matrix A when calculating the final result matrix. However, this does not improve the runtime algorithmatically.

Key Takeaways

* The smart solution does not calculate the final result at once.
* The smart solution skips a For−loop calculation if the value A[i][k] from matrix A is zero.
* Sparse matrix manipulation can help improve the solution, but it does not improve the runtime algorithmatically.

Example Multiple Choice Question

What is the key advantage of the smart solution?

A) It calculates the final result at once.
B) It skips a For−loop calculation if the value A[i][k] from matrix A is zero.
C) It uses sparse matrix manipulation to improve runtime.
D) It has a higher time complexity than the brute force solution.

Answer: B) It skips a For−loop calculation if the value A[i][k] from matrix A is zero.

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A sample document from Finemath-4+

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AB = | −1 0 3 | x | 0 0 0 | = | −7 0 3 |
| 0 0 1 |

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C[i][1]=A[i][0]
B[0][1] + A[i][1]B[1][1] + A[i][2]B[2][1] + ... A[i][k]B[k][0] .... A[i][K]B[K][1]

C[i][nB]=A[i][0]
B[0][nB] + A[i][1]B[1][nB] + A[i][2]B[2][nB] + ... A[i][k]B[k][nB] .... A[i][K]*B[K][nB]

As you can see from above: for the same value A[i][k] from the first matrix, it will be used at most nB times if A[i][k] is not zero. And the smart solution is taking advantage of that!!!, the smart solution can be described as:

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The smart solution code is as follow:

(Credit:@yavinci; I am having a different version of the solution, so I am directly referencing the original version as a reference to demonstrate how mine is different).

Based on the discussion above, the inner checking (if (B[k][j] != 0)) is actually not necessary, because whether or not we have that check, we still iterate nB times, (since the operation C[i][j] += A[i][k] * B[k][j]; inside the if−check is O(1) time)

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A sample document from MegaMath-Web

14 Heat and Transfer Methods

104 14.5 Conduction

Summary

– Calculate thermal conductivity.
– Observe conduction of heat in collisions.
– Study thermal conductivities of common substances.

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 2 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $latex \boldsymbol{\Delta T = T_{\textbf{hot}} T_{\textbf{cold}}} $. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross–sectional area. If you touch a cold wall with your palm , your hand cools faster than if you just touch it with your fingertip.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose thatis greater thanso that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in Figure 3, is given by

where Q/t is the rate of heat transfer in watts or kilocalories per second,is the thermal conductivity of the material,andare its surface area and thickness, as shown in Figure 3, andis the temperature difference across the slab. Table 3 gives representative values of thermal conductivity.

Example 1: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m20.950 m2 and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages atThe inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

– Identify the knowns.
– Identify the unknowns. We need to solve for the mass of the ice,We will also need to solve for the net heat transferred to melt the ice,

– Determine which equations to use. The rate of heat transfer by conduction is given by [latex]\boldsymbol{=}[/latex]
– The heat is used to melt the ice:
– Insert the known values: [latex]\boldsymbol{=}[/latex][latex]\boldsymbol{=\:13.3\textbf{ J/s}}.[/latex]
– Multiply the rate of heat transfer by the time ():
– Set this equal to the heat transferred to melt the ice:Solve for the mass [latex size=2]\boldsymbol{\frac{Q}{L_{\textbf{f}}}}[/latex][latex size=2]\boldsymbol{\frac{1.15\times10^6\textbf{ J}}{334\times10^3\textbf{ J/kg}}}[/latex]

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (710 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose–down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of airs poor thermal conductivity.

Substance	Thermal conductivity k (J/smC)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.080.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Table 3. Thermal Conductivities of Common Substances^{1} |

A combination of material and thickness is often manipulated to develop good insulatorsthe smaller the conductivityand the larger the thicknessthe better. The ratio ofwill thus be large for a good insulator. The ratiois called thefactor. The rate of conductive heat transfer is inversely proportional toThe larger the value ofthe better the insulation.factors are most commonly quoted for household insulation, refrigerators, and the likeunfortunately, it is still in non−metric units of ft^{2}Fh/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 F). A couple of representative values are anfactor of 11 for 3.5−in−thick fiberglass batts (pieces) of insulation and anfactor of 19 for 6.5−in−thick fiberglass batts. Walls are usually insulated with 3.5−in batts, while ceilings are usually insulated with 6.5−in batts. In cold climates, thicker batts may be used in ceilings and walls.

Note that in Table 3, the best thermal conductorssilver, copper, gold, and aluminumare also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 2: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference_{.}

Solution

– Identify the knowns and convert them to the SI units.
The thickness of the pan,the area of the pan,and the thermal conductivity,

– Calculate the necessary heat of vaporization of 1 g of water:
– Calculate the rate of heat transfer given that 1 g of water melts in one second:
– Insert the knowns into the equation and solve for the temperature difference:
[latex size=2]\boldsymbol{\frac{Q}{t}\left(\frac{d}{kA}\right)}[/latex][latex size=2]\boldsymbol{\frac{8.00\times10^{−3}\textbf{ m}}{(220\textbf{ J/s}\cdotp\textbf{m}\cdotp^{\circ}\textbf{C})(1.54\times10^{−2}\textbf{ m}^2)}}[/latex]

Discussion

The value for the heat transferis typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Check Your Understanding

1: How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Summary

– Heat conduction is the transfer of heat between two objects in direct contact with each other.
– The rate of heat transfer(energy per unit time) is proportional to the temperature differenceand the contact areaand inversely proportional to the distancebetween the objects:
[latex]\boldsymbol{=}[/latex]

Conceptual Questions

**1: **Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

**2: **Loose−fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.

Problems & Exercises

**1: **(a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls isand their inside surface is atwhile their outside surface is at(b) How many 1−kW room heaters would be needed to balance the heat transfer due to conduction?

**2: **The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through awindow that isthick (1/4 in) if the temperatures of the inner and outer surfaces areandrespectively. This rapid rate will not be maintainedthe inner surface will cool, and even result in frost formation.

**3: **Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature isthe skin temperature isthe thickness of the tissues between averagesand the surface area is

**4: **Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area ofwith each foot. Both the ceramic and the carpet are 2.00 cm thick and areon their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at

**5: **A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

**6: **(a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00−mm−thick callus with a conductivity at the low end of the range for wood and its density isThe area of contact isthe temperature of the coals isand the time in contact is 1.00 s.

(b) What temperature increase is produced in theof tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

**7: **(a) What is the rate of heat conduction through the 3.00−cm−thick fur of a large animal having asurface area? Assume that the animals skin temperature isthat the air temperature isand that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

**8: **A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed inwater. The walruss internal core temperature isand it has a surface area ofWhat is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

**9: **Compare the rate of heat conduction through a 13.0−cm−thick wall that has an area ofand a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area ofassuming the same temperature difference across each.

**10: **Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is

**11: **Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 2, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

**12: **One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

**13: **(a) Calculate the rate of heat conduction through a double−paned window that has aarea and is made of two panes of 0.800−cm−thick glass separated by a 1.00−cm air gap. The inside surface temperature iswhile that on the outside is(Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60−cm−thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

**14: **Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years .) Suppose you wish to install the extra insulation in Exercise 12. If energy cost $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the averagefor the 120 day heating season to be

**15: **For the human body, what is the rate of heat transfer by conduction through the bodys tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature isand the skin area isHow does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

Footnotes

− 1 At temperatures near 0C.

Glossary

*R*factor− the ratio of thickness to the conductivity of a material

− rate of conductive heat transfer
− rate of heat transfer from one material to another

− thermal conductivity
− the property of a materials ability to conduct heat

Solutions

**Check Your Understanding
**

1: Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubledThe distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

∗∗Problems & Exercises∗∗

∗∗1:∗∗

(a)W

(b) One

∗∗3:∗∗

84.0 W

∗∗5:∗∗

2.59 kg

∗∗7:∗∗

(a) 39.7 W

(b) 820 kcal

∗∗9:∗∗

35 to 1, window to wall

∗∗11:∗∗

∗∗13:∗∗

(a) 83 W

(b) 24 times that of a double pane window.

∗∗15:∗∗

20.0 W, 17.2% of 2400 kcal per day

Meta:

URL: http://pressbooks-dev.oer.hawaii.edu/collegephysics/chapter/14-5-conduction/

WARC Filename: crawl-data/CC-MAIN-2019-04/segments/1547583658844.27/warc/CC-MAIN-20190117062012-20190117084012-00486.warc.gz

Lang score: 0.767403781414032

Math score: 0.5140737295150757

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#16.9 Waves
11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

Introduction

Waves are a fundamental concept in physics, characterized by the transfer of energy through a medium. Understanding waves involves exploring various types, including mechanical and electromagnetic waves, each with distinct properties and behaviors. This section delves into the characteristics and equations governing wave motion.

Wave Properties

A wave is defined by several key properties: wavelength, frequency, amplitude, and speed. The relationship between these properties is given by the wave equation:

\[
v = f \lambda
\]

where \(v \) is the wave speed, \(f \) is the frequency, and \(\lambda \) is the wavelength.

Cohesion and Adhesion in Liquids

Cohesion refers to the attractive forces between molecules of the same substance, while adhesion describes the attraction between molecules of different substances. These forces are crucial in understanding phenomena such as surface tension and capillary action.

Surface Tension

Surface tension arises from cohesive forces among liquid molecules at the liquid’s surface. It is responsible for the formation of droplets and the ability of some insects to walk on water. The force of surface tension can be expressed as:

\[
\gamma = \frac{F}{L}
\]

where \(\gamma \) is the surface tension, \(F \) is the force, and \(L \) is the length over which the force acts.

Capillary Action

Capillary action is the ability of a liquid to flow in narrow spaces without the assistance of external forces. This phenomenon is a result of the interplay between cohesive and adhesive forces. The height to which a liquid will rise in a capillary tube is given by:

\[
h = \frac{2\gamma \cos \theta}{\rho g r}
\]

where \(h \) is the height, \(\gamma \) is the surface tension, \(\theta \) is the contact angle, \(\rho \) is the density of the liquid, \(g \) is the acceleration due to gravity, and \(r \) is the radius of the tube.

Conclusion

The study of waves and the intermolecular forces in liquids provides insight into various natural and technological processes. Understanding these concepts is essential for applications ranging from medical diagnostics to environmental science.

References

– Feynman, R. P., Leighton, R. B., & Sands, M. (1963). ”The Feynman Lectures on Physics, Volume I.” Addison–Wesley.

– Kittel, C., & Knight, W. D. (2005). "Introduction to Solid State Physics." Wiley.

Comments

– "The explanation of surface tension was particularly clear and detailed, making it easier to grasp the concept." – Reader A
– "I appreciate the inclusion of real–world examples to illustrate capillary action." – Reader B

This content is intended to provide a comprehensive overview of waves and the principles of cohesion and adhesion in liquids. It is structured to facilitate understanding and application of these fundamental concepts in physics.

Introduction to College Physics

I. 1 Introduction: The Nature of Science and Physics

1. Introduction to Science and the Realm of Physics, Physical Quantities, and Units

The study of physics is fundamental in understanding the natural world. Physics explores the fundamental principles that govern the universe, from the smallest particles to the largest galaxies. It encompasses the study of physical quantities such as mass, time, and energy, and the units used to measure them. Understanding these concepts is crucial for comprehending more complex phenomena.

2. 1.1 Physics: An Introduction

Physics is the science that deals with the structure of matter and the interactions between the fundamental constituents of the observable universe. The study of physics involves understanding the laws that govern these interactions and applying them to solve problems in various fields.

3. 1.2 Physical Quantities and Units

Physical quantities are properties of objects that can be measured and quantified. They include concepts such as length, mass, time, and temperature. Units are the standard measurements used to express these quantities, such as meters for length, kilograms for mass, seconds for time, and Kelvin for temperature.

4. 1.3 Accuracy, Precision, and Significant Figures

Accuracy refers to how close a measured value is to the true value, while precision refers to the consistency of repeated measurements. Significant figures are used to express the precision of a measurement, indicating the number of meaningful digits in a number.

5. 1.4 Approximation

Approximation involves estimating values to simplify calculations or to provide a reasonable degree of accuracy. It is a useful technique in physics when exact values are not necessary or feasible to obtain.

II. 2 Kinematics

6. Introduction to One–Dimensional Kinematics

Kinematics is the study of motion without considering the forces that cause it. One–dimensional kinematics focuses on motion along a straight line and involves concepts such as displacement, velocity, and acceleration.

7. 2.1 Displacement

Displacement is the change in position of an object. It is a vector quantity, meaning it has both magnitude and direction. It is represented as $\Delta x = x_f - x_i$, where x_f is the final position and x_i is the initial position.

8. 2.2 Vectors, Scalars, and Coordinate Systems

Vectors are quantities that have both magnitude and direction, such as displacement, velocity, and acceleration. Scalars are quantities that have only magnitude, such as distance, speed, and time. Coordinate systems are used to define the position of points in space, with common systems including Cartesian, polar, and spherical coordinates.

9. 2.3 Time, Velocity, and Speed

Time is a scalar quantity that measures the duration of an event. Velocity is a vector quantity that describes the rate of change of displacement, given by $v = \frac{\Delta x}{\Delta t}$. Speed is a scalar quantity that describes the rate of change of distance, given by $\text{speed} = \frac{\Delta s}{\Delta t}$.

10. 2.4 Acceleration

Acceleration is the rate of change of velocity. It is a vector quantity given by $a = \frac{\Delta v}{\Delta t}$. Acceleration can be caused by changes in speed or direction.

11. 2.5 Motion Equations for Constant Acceleration in One Dimension

For an object moving with constant acceleration, the following kinematic equations can be used:

1. $v = v_i + at$
2. $x = x_i + v_i t + \frac{1}{2}at^2$
3. $v^2 = v_i^2 + 2a(x - x_i)$

where v_i is the initial velocity, v is the final velocity, a is the acceleration, t is the time, x_i is the initial position, and x is the final position.

12. 2.6 Problem−Solving Basics for One−Dimensional Kinematics

To solve kinematics problems, it is essential to:

1. Identify the known and unknown quantities.
2. Choose the appropriate kinematic equation.
3. Solve for the unknown quantity.
4. Check the units and reasonableness of the answer.

13. 2.7 Falling Objects

Falling objects are subject to the acceleration due to gravity, $g \approx 9.81 \, \text{m/s}^2$. The motion of falling objects can be analyzed using the kinematic equations, assuming air resistance is negligible.

14. 2.8 Graphical Analysis of One−Dimensional Motion

Graphs of position vs. time, velocity vs. time, and acceleration vs. time can provide valuable insights into the motion of an object. The slope of a position−time graph represents velocity, while the slope of a velocity−time graph represents acceleration.

III. 3 Two−dimensional Kinematics

15. Introduction to Two−Dimensional Kinematics

Two−dimensional kinematics involves the study of motion in a plane, where both the x and y components of motion are considered. This requires analyzing the motion in each direction separately and then combining the results.

16. 3.1 Kinematics in Two Dimensions: An Introduction

In two−dimensional kinematics, motion can be described by its x and y components. The position, velocity, and acceleration vectors can be broken down into these components, allowing for the analysis of motion in each direction independently.

17. 3.2 Vector Addition and Subtraction: Graphical Methods

Vector addition and subtraction can be performed graphically using the head−to−tail method or the parallelogram method. These methods involve drawing vectors to scale and using geometric constructions to find the resultant vector.

18. 3.3 Vector Addition and Subtraction: Analytical Methods

Analytically, vectors can be added or subtracted by breaking them down into their components. The x and y components of the resultant vector are found by summing the corresponding components of the individual vectors.

For two vectors $\mathbf{A}$ and $\mathbf{B}$:

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y$$

The magnitude of the resultant vector $\mathbf{R}$ is given by:

$$R = \sqrt{R_x^2 + R_y^2}$$

And the direction (angle θ) is given by:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

19. 3.4 Projectile Motion

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The horizontal and vertical components of motion are analyzed separately, with the horizontal motion being constant velocity and the vertical motion being uniformly accelerated motion.

The equations for projectile motion are:

Horizontal motion:
$$x = v_{ix} t$$

Vertical motion:
$$y = v_{iy} t - \frac{1}{2} g t^2$$

The initial velocity components are:
$$v_{ix} = v_i \cos(\theta)$$
$$v_{iy} = v_i \sin(\theta)$$

20. 3.5 Addition of Velocities

The addition of velocities involves combining the velocity vectors of different reference frames. The relative velocity of an object with respect to a reference frame is found by vector addition of the object's velocity relative to another frame and the velocity of that frame relative to the reference frame.

If $\mathbf{v}_{AB}$ is the velocity of object A relative to B, and $\mathbf{v}_{BC}$ is the velocity of B relative to C, then the velocity of A relative to C is:

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$$

IV. 4 Dynamics: Force and Newton's Laws of Motion

21. Introduction to Dynamics: Newtons Laws of Motion

Dynamics is the study of forces and their effects on motion. Newton's laws of motion form the foundation of classical mechanics and describe the relationship between the motion of an object and the forces acting on it.

22. 4.1 Development of Force Concept

The concept of force was developed to explain the interactions between objects that cause changes in motion. A force is a vector quantity that can cause an object to accelerate, decelerate, or change direction.

23. 4.2 Newtons First Law of Motion: Inertia

Newton’s first law of motion, also known as the law of inertia, states that an object at rest will remain at rest, and an object in motion will continue in motion with a constant velocity, unless acted upon by a net external force.

24. 4.3 Newtons Second Law of Motion: Concept of a System

Newton’s second law of motion states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. It is expressed as:

$$\mathbf{F}_{\text{net}} = m \mathbf{a}$$

where $\mathbf{F}_{\text{net}}$ is the net force, m is the mass, and $\mathbf{a}$ is the acceleration.

25. 4.5 Normal, Tension, and Other Examples of Forces

Normal force is the perpendicular force exerted by a surface on an object in contact with it. Tension is the force transmitted through a string, rope, or cable when it is pulled tight by forces acting from opposite ends.

Other examples of forces include gravitational force, frictional force, and applied force.

26. 4.4 Newtons Third Law of Motion: Symmetry in Forces

Newton’s third law of motion states that for every action, there is an equal and opposite reaction. This means that forces always occur in pairs; if object A exerts a force on object B, then object B exerts an equal and opposite force on object A.

27. 4.6 Problem−Solving Strategies

To solve dynamics problems involving forces and Newton’s laws, follow these steps:

1. Draw a free−body diagram to identify all the forces acting on the object.
2. Apply Newton’s second law ($\mathbf{F}_{\text{net}} = m \mathbf{a}$) in the appropriate direction(s).
3. Solve the resulting equations for the unknown quantities.
4. Check the units and reasonableness of the answer.

28. 4.7 Further Applications of Newtons Laws of Motion

Newton’s laws can be applied to a wide range of problems, including motion on inclined planes, circular motion, and systems of connected objects. Understanding these applications allows for the analysis of more complex systems and interactions.

29. 4.8 Extended Topic: The Four Basic ForcesAn Introduction

The four fundamental forces in nature are:

1. Gravitational force: The attractive force between masses.
2. Electromagnetic force: The force between charged particles.
3. Strong nuclear force: The force that holds protons and neutrons together in the nucleus.
4. Weak nuclear force: The force responsible for radioactive decay and nuclear fusion.

These forces govern the interactions between particles and are the basis for understanding the behavior of matter and energy.

V. 5 Further Applications of Newton’s Laws: Friction, Drag, and Elasticity

30. Introduction: Further Applications of Newtons Laws

Newton’s laws of motion can be extended to analyze the effects of friction, drag, and elasticity on the motion of objects.

31. 5.1 Friction

Friction is the force that opposes the relative motion or tendency of such motion between two surfaces in contact. It can be categorized into static friction (preventing motion) and kinetic friction (opposing motion).

The frictional force f is given by:

$$ f = \mu N $$

where μ is the coefficient of friction and N is the normal force.

32. 5.2 Drag Forces

Drag is the force exerted by a fluid (such as air or water) on an object moving through it. It acts in the direction opposite to the object’s motion and depends on the object’s velocity, shape, and the fluid’s properties.

The drag force F_d can be expressed as:

$$ F_d = \frac{1}{2} C_d \rho A v^2 $$

where C_d is the drag coefficient, ρ is the fluid density, A is the cross-sectional area, and v is the velocity.

33. 5.3 Elasticity: Stress and Strain

Elasticity is the property of a material to return to its original shape after being deformed. Stress is the force per unit area applied to a material, while strain is the deformation per unit length.

The relationship between stress (σ) and strain (ϵ) is given by Hooke’s law for elastic materials:

$$ \sigma = E \epsilon $$

where E is the Young’s modulus of the material.

VI. 6 Uniform Circular Motion and Gravitation

34. Introduction to Uniform Circular Motion and Gravitation

Uniform circular motion involves an object moving in a circular path with a constant speed. Gravitation is the attractive force between masses, described by Newton’s law of universal gravitation.

35. 6.1 Rotation Angle and Angular Velocity

The rotation angle θ is the angle through which an object rotates. Angular velocity ω is the rate of change of the rotation angle, given by:

$$ \omega = \frac{\Delta \theta}{\Delta t} $$

36. 6.2 Centripetal Acceleration

Centripetal acceleration is the acceleration of an object moving in a circular path, directed towards the center of the circle. It is given by:

$$ a_c = \frac{v^2}{r} $$

where v is the linear speed and r is the radius of the circle.

37. 6.3 Centripetal Force

Centripetal force is the net force causing the centripetal acceleration of an object in circular motion. It is directed towards the center of the circle and is given by:

$$ F_c = m a_c = \frac{m v^2}{r} $$

38. 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

In non-inertial reference frames (accelerating frames), fictitious forces must be introduced to explain the motion of objects. The Coriolis force is a fictitious force experienced by objects in a rotating reference frame, given by:

$$ \mathbf{F}_{\text{Coriolis}} = -2m (\mathbf{\omega} \times \mathbf{v}) $$

where $\mathbf{\omega}$ is the angular velocity of the rotating frame and $\mathbf{v}$ is the velocity of the object.

39. 6.5 Newtons Universal Law of Gravitation

Newton's universal law of gravitation states that every mass attracts every other mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. It is given by:

$$ F_g = G \frac{m_1 m_2}{r^2} $$

where F_g is the gravitational force, G is the gravitational constant, m_1 and m_2 are the masses, and r is the distance between the centers of the masses.

40. 6.6 Satellites and Keplers Laws: An Argument for Simplicity

Kepler's laws describe the motion of planets and satellites. They are:

1. The orbit of a planet is an ellipse with the Sun at one focus.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

These laws provide a simple description of the motion of celestial bodies and are consistent with Newton's law of gravitation.

VII. 7 Work, Energy, and Energy Resources

41. Introduction to Work, Energy, and Energy Resources

Work, energy, and energy resources are fundamental concepts in physics. Work is the transfer of energy by a force acting over a distance, while energy is the capacity to do work. Energy resources are sources of energy used to perform work.

42. 7.1 Work: The Scientific Definition

Work is done when a force causes a displacement of an object. The scientific definition of work W is given by:

$$ W = \mathbf{F} \cdot \mathbf{d} = F d \cos(\theta) $$

where $\mathbf{F}$ is the force, $\mathbf{d}$ is the displacement, and θ is the angle between the force and displacement vectors.

43. 7.2 Kinetic Energy and the Work-Energy Theorem

Kinetic energy (KE) is the energy of an object due to its motion, given by:

$$ KE = \frac{1}{2} m v^2 $$

The work–energy theorem states that the work done on an object is equal to the change in its kinetic energy:

$$W = \Delta KE = KE_f - KE_i$$

where KE_f is the final kinetic energy and KE_i is the initial kinetic energy.

44. 7.3 Gravitational Potential Energy

Gravitational potential energy (PE_g) is the energy an object possesses due to its position in a gravitational field. It is given by:

$$PE_g = mgh$$

where m is the mass, g is the acceleration due to gravity, and h is the height above a reference point.

45. 7.4 Conservative Forces and Potential Energy

A conservative force is a force for which the work done is independent of the path taken and depends only on the initial and final positions. Examples include gravitational force and spring force. The potential energy associated with a conservative force is a function of position.

46. 7.5 Nonconservative Forces

Nonconservative forces are forces for which the work done depends on the path taken. Examples include friction and air resistance. The work done by nonconservative forces results in the dissipation of mechanical energy as heat or other forms of energy.

47. 7.6 Conservation of Energy

The principle of conservation of energy states that the total energy of an isolated system remains constant. Energy can be transformed from one form to another, but the total energy is conserved .

48. 7.7 Power

Power is the rate at which work is done or energy is transferred. It is given by:

$$P = \frac{W}{t} = Fv$$

where P is the power, W is the work done, t is the time, F is the force, and v is the velocity.

49. 7.8 Work, Energy, and Power in Humans

In humans, work, energy, and power are involved in various activities, from walking and running to lifting objects. The efficiency of energy conversion in the human body affects the amount of work that can be performed.

50. 7.9 World Energy Use

World energy use involves the consumption of various energy resources, including fossil fuels, nuclear energy, and renewable sources. Understanding energy use is crucial for addressing environmental and sustainability challenges.

VIII. 8 Linear Momentum and Collisions

51. Introduction to Linear Momentum and Collisions

Linear momentum is a measure of the motion of an object and is the product of its mass and velocity. Collisions involve the interaction of objects and the transfer of momentum.

52. 8.1 Linear Momentum and Force

Linear momentum ($\mathbf{p}$) is given by:

$$\mathbf{p} = m \mathbf{v}$$

where m is the mass and $\mathbf{v}$ is the velocity. The force acting on an object is related to the rate of change of its momentum:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

53. 8.2 Impulse

Impulse is the change in momentum of an object due to a force acting over a time interval. It is given by:

$$\mathbf{J} = \mathbf{F} \Delta t = \Delta \mathbf{p}$$

where $\mathbf{J}$ is the impulse, $\mathbf{F}$ is the force, Δt is the time interval, and $\Delta \mathbf{p}$ is the change in momentum.

54. 8.3 Conservation of Momentum

The principle of conservation of momentum states that the total momentum of an isolated system remains constant if no external forces act on it. For a system of particles, the total momentum before an interaction is equal to the total momentum after the interaction:

$$\sum \mathbf{p}_{\text{initial}} = \sum \mathbf{p}_{\text{final}}$$

55. 8.4 Elastic Collisions in One Dimension

In an elastic collision, both momentum and kinetic energy are conserved. For two objects with masses m_1 and m_2, and initial velocities v_{1i} and v_{2i}, the final velocities v_{1f} and v_{2f} after an elastic collision are given by:

$$v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2 v_{2i}}{m_1 + m_2}$$

$$v_{2f} = \frac{(m_2 - m_1)v_{2i} + 2m_1 v_{1i}}{m_1 + m_2}$$

56. 8.5 Inelastic Collisions in One Dimension

In an inelastic collision, momentum is conserved, but kinetic energy is not. If two objects stick together after the collision, the final velocity v_f is given by:

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

57. 8.6 Collisions of Point Masses in Two Dimensions

In two-dimensional collisions, both the x and y components of momentum are conserved. The conservation of momentum equations for a system of two objects are:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

58. 8.7 Introduction to Rocket Propulsion

Rocket propulsion involves the expulsion of mass (exhaust) to generate thrust according to Newton’s third law of motion. The change in momentum of the rocket is equal and opposite to the change in momentum of the expelled mass.

IX. 9 Statics and Torque

59. Introduction to Statics and Torque

Statics is the study of forces in equilibrium, where the net force and net torque on an object are zero. Torque is the measure of the rotational effect of a force.

60. 9.1 The First Condition for Equilibrium

The first condition for equilibrium states that the net force acting on an object must be zero:

$$\sum \mathbf{F} = 0$$

61. 9.2 The Second Condition for Equilibrium

The second condition for equilibrium states that the net torque acting on an object must be zero:

$$\sum \tau = 0$$

where τ is the torque, given by:

$$\tau = r \times \mathbf{F} = r F \sin(\theta)$$

where r is the lever arm (distance from the axis of rotation to the point where the force is applied), $\mathbf{F}$ is the force, and θ is the angle between $\mathbf{F}$ and r.

62. 9.3 Stability

The stability of an object in equilibrium depends on its center of gravity and the base of support. An object is stable if it returns to its equilibrium position after a small displacement.

63. 9.4 Applications of Statics, Including Problem−Solving Strategies

Statics has many applications, including the analysis of structures, machines, and mechanical systems. Problem−solving strategies in statics involve:

1. Drawing a free−body diagram.
2. Applying the equilibrium conditions ($\sum \mathbf{F} = 0$ and $\sum \tau = 0$).
3. Solving the resulting equations for the unknown forces or torques.
4. Checking the units and reasonableness of the answer.

64. 9.5 Simple Machines

Simple machines are devices that make work easier by changing the direction or magnitude of a force. Examples include levers, pulleys, and inclined planes.

65. 9.6 Forces and Torques in Muscles and Joints

The human body uses muscles and joints as simple machines to generate force and torque. Understanding the forces and torques in muscles and joints is important for analyzing human movement and designing assistive devices.

X. 10 Rotational Motion and Angular Momentum

66. Introduction to Rotational Motion and Angular Momentum

Rotational motion involves the rotation of an object around an axis. Angular momentum is the rotational analog of linear momentum and is conserved in isolated systems.

67. 10.1 Angular Acceleration

Angular acceleration (α) is the rate of change of angular velocity (ω), given by:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

68. 10.2 Kinematics of Rotational Motion

The kinematic equations for rotational motion are analogous to those for linear motion. They relate angular displacement (θ), angular velocity (ω), angular acceleration (α), and time (t):

1. $\theta = \omega_i t + \frac{1}{2} \alpha t^2$
2. $\omega_f = \omega_i + \alpha t$
3. $\omega_f^2 = \omega_i^2 + 2 \alpha \theta$

where ω_i is the initial angular velocity and ω_f is the final angular velocity.

69. 10.3 Dynamics of Rotational Motion: Rotational Inertia

Rotational inertia (moment of inertia) is the rotational analog of mass and measures an object's resistance to changes in its rotational motion. It is given by:

$$I = \sum m_i r_i^2$$

where m_i is the mass of a point in the object and r_i is the distance from the axis of rotation.

70. 10.4 Rotational Kinetic Energy: Work and Energy Revisited

Rotational kinetic energy (KE_{rot}) is the energy of an object due to its rotation, given by:

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

where I is the moment of inertia and ω is the angular velocity.

71. 10.5 Angular Momentum and Its Conservation

Angular momentum (L) is the product of the moment of inertia and the angular velocity:

$$L = I \omega$$

The principle of conservation of angular momentum states that the total angular momentum of an isolated system remains constant if no external torques act on it:

$$\sum L_{\text{initial}} = \sum L_{\text{final}}$$

72. 10.6 Collisions of Extended Bodies in Two Dimensions

In collisions involving extended bodies, both linear and angular momentum are conserved. The conservation of linear momentum and angular momentum must be applied to analyze the motion of the bodies before and after the collision.

73. 10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

Gyroscopic effects arise from the vector nature of angular momentum. When a torque is applied to a spinning object, it experiences a change in angular momentum that is perpendicular to both the torque and the initial angular momentum, causing the object to precess.

XI. 11 Fluid Statics

74. Introduction to Fluid Statics

Fluid statics is the study of fluids at rest and the forces and pressures exerted by and on them.

75. 11.1 What Is a Fluid?

A fluid is a substance that can flow and does not have a fixed shape. Fluids include liquids and gases, which can deform under the action of shear stress.

76. 11.2 Density

Density (ρ) is the mass per unit volume of a substance, given by:

$$\rho = \frac{m}{V}$$

where m is the mass and V is the volume.

77. 11.3 Pressure

Pressure (P) is the force exerted per unit area, given by:

$$P = \frac{F}{A}$$

where F is the force and A is the area over which the force is applied.

78. 11.4 Variation of Pressure with Depth in a Fluid

In a fluid at rest, the pressure increases with depth due to the weight of the fluid above. The pressure at a depth h in a fluid of density ρ is given by:

$$P = P_0 + \rho g h$$

where P_0 is the pressure at the surface, ρ is the fluid density, g is the acceleration due to gravity, and h is the depth.

79. 11.5 Pascals Principle

Pascal’s principle states that a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and the walls of its container.

80. 11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

Gauge pressure is the pressure relative to atmospheric pressure. Absolute pressure is the total pressure, including atmospheric pressure. Pressure measurement devices, such as manometers and barometers, are used to measure pressure.

81. 11.7 Archimedes Principle

Archimedes’ principle states that a body immersed in a fluid experiences a buoyant force equal to the weight of the fluid displaced by the body. The buoyant force (F_b) is given by:

$$F_b = \rho_f V_f g$$

where ρ_f is the fluid density, V_f is the volume of the fluid displaced, and g is the acceleration due to gravity.

82. 11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

Cohesion is the attraction between molecules of the same substance, while adhesion is the attraction between molecules of different substances. Surface tension is the result of cohesive forces and causes the surface of a liquid to contract. Capillary action is the ability of a liquid to flow in narrow spaces due to adhesive forces.

83. 11.9 Pressures in the Body

Pressures in the body are important for various physiological processes, including blood circulation and respiration. Understanding these pressures is crucial for diagnosing and treating medical conditions.

XII. 12 Fluid Dynamics and Its Biological and Medical Applications

84. Introduction to Fluid Dynamics and Its Biological and Medical Applications

Fluid dynamics is the study of the motion of fluids and the forces acting on them. It has important applications in biology and medicine, such as blood flow and respiratory mechanics.

85. 12.1 Flow Rate and Its Relation to Velocity

Flow rate (Q) is the volume of fluid passing through a cross−section per unit time, given by:

$$Q = A v$$

where A is the cross−sectional area and v is the fluid velocity.

86. 12.2 Bernoullis Equation

Bernoulli's equation relates the pressure, velocity, and height of a fluid in steady, incompressible flow. It is given by:

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

where P is the pressure, ρ is the fluid density, v is the fluid velocity, g is the acceleration due to gravity, and h is the height above a reference point.

87. 12.3 The Most General Applications of Bernoullis Equation

Bernoulli's equation has various applications, including the analysis of fluid flow in pipes, the lift on an airplane wing, and the operation of a Venturi meter.

88. 12.4 Viscosity and Laminar Flow; Poiseuilles Law

Viscosity is the measure of a fluid's resistance to flow. Laminar flow is characterized by smooth, orderly fluid motion. Poiseuille's law describes the flow rate of a viscous fluid through a cylindrical pipe:

$$Q = \frac{\pi r^4 \Delta P}{8 \eta L}$$

where r is the pipe radius, ΔP is the pressure difference, η is the fluid viscosity, and L is the pipe length.

89. 12.5 The Onset of Turbulence

Turbulence is characterized by chaotic fluid motion and occurs when the Reynolds number (Re) exceeds a critical value. The Reynolds number is given by:

$$Re = \frac{\rho v L}{\eta}$$

where ρ is the fluid density, v is the fluid velocity, L is a characteristic length, and η is the fluid viscosity.

90. 12.6 Motion of an Object in a Viscous Fluid

The motion of an object in a viscous fluid is affected by the drag force, which depends on the fluid's viscosity and the object's shape and velocity. The drag force (F_d) is given by:

$$F_d = \frac{1}{2} C_d \rho A v^2$$

where C_d is the drag coefficient, ρ is the fluid density, A is the cross-sectional area, and v is the velocity.

91. 12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

Molecular transport phenomena involve the movement of molecules within fluids. Diffusion is the movement of molecules from an area of high concentration to an area of low concentration. Osmosis is the movement of water across a semipermeable membrane from a region of low solute concentration to a region of high solute concentration.

XIII. 13 Temperature, Kinetic Theory, and the Gas Laws

92. Introduction to Temperature, Kinetic Theory, and the Gas Laws

Temperature is a measure of the average kinetic energy of the particles in a substance. The kinetic theory of gases explains the behavior of gases in terms of the motion of their molecules. The gas laws describe the relationships between the pressure, volume, temperature, and number of moles of a gas.

93. 13.1 Temperature

Temperature is measured on scales such as Celsius, Fahrenheit, and Kelvin. The Kelvin scale is the absolute temperature scale, with 0 K being absolute zero.

94. 13.2 Thermal Expansion of Solids and Liquids

Thermal expansion is the increase in size of a substance with temperature. The linear expansion of a solid is given by:

$$ \Delta L = \alpha L \Delta T $$

where ΔL is the change in length, α is the coefficient of linear expansion, L is the original length, and ΔT is the change in temperature.

The volumetric expansion of a liquid is given by:

$$ \Delta V = \beta V \Delta T $$

where ΔV is the change in volume, β is the coefficient of volumetric expansion, V is the original volume, and ΔT is the change in temperature.

95. 13.3 The Ideal Gas Law

The ideal gas law relates the pressure, volume, temperature, and number of moles of an ideal gas:

$$ PV = nRT $$

where P is the pressure, V is the volume, n is the number of moles, R is the ideal gas constant, and T is the temperature in Kelvin.

96. 13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

The kinetic theory of gases explains pressure and temperature in terms of the motion of gas molecules. The pressure exerted by a gas is due to collisions of its molecules with the walls of the container. The temperature of a gas is related to the average kinetic energy of its molecules:

$$ \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T $$

where m is the mass of a molecule, $\langle v^2 \rangle$ is the mean square velocity, k_B is the Boltzmann constant, and T is the temperature in Kelvin.

97. 13.5 Phase Changes

Phase changes involve the transition of a substance from one state of matter to another, such as solid to liquid (melting), liquid to gas (vaporization), or solid to gas (sublimation). These processes involve changes in energy and occur at specific temperatures and pressures.

98. 13.6 Humidity, Evaporation, and Boiling

Humidity is the amount of water vapor in the air. Evaporation is the process by which molecules at the surface of a liquid gain enough energy to escape into the gas phase. Boiling is the rapid vaporization of a liquid when its vapor pressure equals the external pressure.

XIV. 14 Heat and Transfer Methods

99. Introduction to Heat and Heat Transfer Methods

Heat is the transfer of thermal energy between systems due to a temperature difference. Heat transfer methods include conduction, convection, and radiation.

100. 14.1 Heat

Heat (Q) is the energy transferred due to a temperature difference, given by:

$$ Q = mc\Delta T $$

where m is the mass, c is the specific heat capacity, and ΔT is the change in temperature.

101. 14.2 Temperature Change and Heat Capacity

The temperature change of a substance is related to the amount of heat transferred to it and its heat capacity. The heat capacity (C) is the amount of heat required to change the temperature of an object by one degree Celsius:

$$Q = C \Delta T$$

102. 14.3 Phase Change and Latent Heat

Phase changes involve the absorption or release of latent heat, which is the heat required to change the phase of a substance without changing its temperature. The latent heat of fusion (L_f) is the heat required to change a substance from solid to liquid, and the latent heat of vaporization (L_v) is the heat required to change a substance from liquid to gas:

$$Q = mL_f$$
$$Q = mL_v$$

103. 14.4 Heat Transfer Methods

Heat transfer occurs through conduction, convection, and radiation:

- **Conduction** is the transfer of heat through a material without the movement of the material itself. The rate of heat transfer by conduction is given by:

  $$\frac{Q}{t} = kA \frac{\Delta T}{d}$$

  where $\frac{Q}{t}$ is the rate of heat transfer, k is the thermal conductivity, A is the cross-sectional area, ΔT is the temperature difference, and d is the thickness of the material.

- **Convection** is the transfer of heat by the movement of a fluid (liquid or gas). The rate of heat transfer by convection is given by:

  $$\frac{Q}{t} = hA \Delta T$$

  where $\frac{Q}{t}$ is the rate of heat transfer, h is the convective heat transfer coefficient, A is the surface area, and ΔT is the temperature difference between the fluid and the surface.

- **Radiation** is the transfer of heat in the form of electromagnetic waves. The rate of heat transfer by radiation is given by the Stefan-Boltzmann law:

  $$\frac{Q}{t} = \sigma e A (T^4 - T_0^4)$$

  where $\frac{Q}{t}$ is the rate of heat transfer, σ is the Stefan-Boltzmann constant, e is the emissivity of the surface, A is the surface area, T is the temperature of the surface, and T_0 is the temperature of the surroundings.

104. 14.5 Conduction

Conduction is the transfer of heat through a material due to the collision of molecules. The rate of heat transfer by conduction is proportional to the temperature gradient and the thermal conductivity of the material.

105. 14.6 Convection

Convection is the transfer of heat by the movement of a fluid. It can be natural (driven by buoyancy forces) or forced (driven by external means such as a fan or pump).

106. 14.7 Radiation

Radiation is the transfer of heat in the form of electromagnetic waves. All objects emit thermal radiation, and the amount of radiation emitted depends on the object's temperature and emissivity.

XV. 15 Thermodynamics

107. Introduction to Thermodynamics

Thermodynamics is the study of the relationships between heat, work, temperature, and energy. It involves the analysis of energy transformations and the laws governing these processes.

108. 15.1 The First Law of Thermodynamics

The first law of thermodynamics, also known as the law of energy conservation, states that the total energy of an isolated system is constant. Energy can be transferred between the system and its surroundings in the form of heat (Q) or work (W), but it cannot be created or destroyed:

$$\Delta U = Q - W$$

where ΔU is the change in internal energy of the system.

109. 15.2 The First Law of Thermodynamics and Some Simple Processes

The first law of thermodynamics can be applied to various thermodynamic processes, such as isothermal (constant temperature), isobaric (constant pressure), isochoric (constant volume), and adiabatic (no heat exchange) processes.

110. 15.3 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

The second law of thermodynamics states that the entropy of an isolated system always increases or remains constant. It introduces the concept of irreversibility and sets limits on the efficiency of heat engines, which convert heat into work.

111. 15.4 Carnots Perfect Heat Engine: The Second Law of Thermodynamics Restated

Carnot’s theorem states that no heat engine can be more efficient than a Carnot engine, which operates between two heat reservoirs at different temperatures. The efficiency (η) of a Carnot engine is given by:

$$\eta = 1 - \frac{T_C}{T_H}$$

where T_C is the temperature of the cold reservoir and T_H is the temperature of the hot reservoir.

112. 15.5 Applications of Thermodynamics: Heat Pumps and Refrigerators

Heat pumps and refrigerators are applications of thermodynamics that transfer heat from a cold region to a hot region. They operate on the same principles as heat engines but in reverse.

113. 15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

Entropy is a measure of the disorder or randomness of a system. The second law of thermodynamics states that the total entropy of an isolated system always increases or remains constant. Entropy also represents the unavailability of energy to do work.

114. 15.7 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

The statistical interpretation of entropy is based on the number of microscopic configurations corresponding to a macroscopic state. The second law of thermodynamics can be understood as the tendency of systems to evolve towards states with the highest number of configurations.

XVI. 16 Oscillatory Motion and Waves

115. Introduction to Oscillatory Motion and Waves

Oscillatory motion involves the repetitive back–and–forth movement of an object about an equilibrium position. Waves are disturbances that transfer energy from one point to another without the transfer of matter.

116. 16.1 Hookes Law: Stress and Strain Revisited

Hooke's law states that the force required to extend or compress a spring is proportional to the displacement from its equilibrium position:

$$F = -kx$$

where F is the force, k is the spring constant, and x is the displacement.

117. 16.2 Period and Frequency in Oscillations

The period (T) is the time taken for one complete cycle of oscillation, and the frequency (f) is the number of cycles per unit time. They are related by:

$$f = \frac{1}{T}$$

118. 16.3 Simple Harmonic Motion: A Special Periodic Motion

Simple harmonic motion (SHM) is a type of periodic motion where the restoring force is proportional to the displacement and directed towards the equilibrium position. The displacement (x) in SHM is given by:

$$x(t) = A \cos(\omega t + \phi)$$

where A is the amplitude, ω is the angular frequency, t is the time, and ϕ is the phase constant.

119. 16.4 The Simple Pendulum

A simple pendulum consists of a mass suspended from a fixed point by a string or rod. The period (T) of a simple pendulum is given by:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity.

120. 16.5 Energy in Simple Harmonic Motion

The energy in simple harmonic motion is the sum of the kinetic energy (KE) and potential energy (PE). The total energy (E) is constant and given by:

$$E = \frac{1}{2} k A^2$$

where k is the spring constant and A is the amplitude.

121. 16.6 Uniform Circular Motion and Simple Harmonic Motion

Simple harmonic motion can be related to uniform circular motion. The projection of uniform circular motion on one axis results in simple harmonic motion.

122. 16.7 D

14.5 Conduction

Summary

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.

The figure shows an insulated wooden partition in a house. The partition is insulated because it encapsulates a cloth–type material.

Figure 1. Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors.

Figure 2. The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high−temperature regions to low−temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

Figure 3. Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1. The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference $T_2 - T_1$, and the substances conductivity k. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material , such as the one in Figure 3, is given by

$$
\frac{Q}{t} = \frac{kA(T_2-T_1)}{d},
$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k is the thermal conductivity of the material, A and d are its surface area and thickness, as shown in Figure 3, and (T_2-T_1) is the temperature difference across the slab.

Example 1: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at $0^{\circ}\textbf{C}$. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at $35.0^{\circ}\textbf{C}$?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

$$A = 0.950\,\text{m}^2; \, d = 2.50\,\text{cm} = 0.0250\,\text{m}; \, T_1 = 0^{\circ}\text{C}; \, T_2 = 35.0^{\circ}\text{C}; \, t = 1\,\text{day} = 24\,\text{hours} = 86{,}400\,\text{s}.$$

2. Identify the unknowns. We need to solve for the mass of the ice, m. We will also need to solve for the net heat transferred to melt the ice, Q.

3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{kA(T_2-T_1)}{d}.$$

4. The heat is used to melt the ice: $Q = mL_{\textbf{f}}$.

5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010\,\text{J/s}\cdot\text{m}\cdot^{\circ}\text{C})(0.950\,\text{m}^2)(35.0^{\circ}\text{C}-0^{\circ}\text{C})}{0.0250\,\text{m}} = 13.3\,\text{J/s}.$$

6. Multiply the rate of heat transfer by the time ($1\,\text{day} = 86{,}400\,\text{s}$):

$$Q = (Q/t)t = (13.3\,\text{J/s})(86{,}400\,\text{s}) = 1.15 \times 10^6\,\text{J}.$$

7. Set this equal to the heat transferred to melt the ice: $Q = mL_{\textbf{f}}$. Solve for the mass m:

$$m = \frac{Q}{L_{\textbf{f}}} = \frac{1.15 \times 10^6\,\text{J}}{334 \times 10^3\,\text{J/kg}} = 3.44\,\text{kg}.$$

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (710 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose−down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of airs poor thermal conductivity.

Substance Thermal Conductivity

Substance	Thermal conductivity k (J/smC)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.080.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042

Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Table 3. Thermal Conductivities of Common Substances

A combination of material and thickness is often manipulated to develop good insulatorsthe smaller the conductivity k and the larger the thickness d, the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the R factor. The rate of conductive heat transfer is inversely proportional to R. The larger the value of R, the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the likeunfortunately, it is still in non−metric units of ftFh/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 F). A couple of representative values are an R factor of 11 for 3.5−in−thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5−in−thick fiberglass batts. Walls are usually insulated with 3.5−in batts, while ceilings are usually insulated with 6.5−in batts. In cold climates, thicker batts may be used in ceilings and walls.

Figure 4. The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in Table 3, the best thermal conductorssilver, copper, gold, and aluminumare also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 2: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The saucepan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

$$
T_2-T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right).
$$

Solution

1. Identify the knowns and convert them to the SI units.
   - The thickness of the pan, $d = 0.800\,\text{cm} = 8.0 \times 10^{-3}\,\text{m}$,
   - The area of the pan, $A = \pi(0.14/2)^2\,\text{m}^2 = 1.54 \times 10^{-2}\,\text{m}^2$,
   - The thermal conductivity, $k = 220\,\text{J/s}\cdot\text{m}\cdot^{\circ}\text{C}$.

2. Calculate the necessary heat of vaporization of 1 g of water:
$$
Q = mL_{\textbf{v}} = (1.00 \times 10^{-3}\,\text{kg})(2256 \times 10^3\,\text{J/kg}) = 2256\,\text{J}.
$$

3. Calculate the rate of heat transfer given that 1 g of water evaporates in one second:
$$
Q/t = 2256\,\text{J/s or }2.26\,\text{kW}.
$$

4. Insert the knowns into the equation and solve for the temperature difference:

$$
T_2-T_1 = \frac{Q}{t}\left(\frac{d}{kA}\right) = (2256\,\text{J/s})\frac{8.00 \times 10^{-3}\,\text{m}}{(220\,\text{J/s}\cdot\text{m}\cdot^{\circ}\text{C})(1.54 \times 10^{-2}\,\text{m}^2)} = 5.33^{\circ}\text{C}.
$$

Discussion

The value for the heat transfer $Q/t = 2.26\,\text{kW or }2256\,\text{J/s}$ is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly 100°C because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Check Your Understanding

1: How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Summary

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference T_2-T_1 and the contact area A and inversely proportional to the distance d between the objects:

$$
\frac{Q}{t} = \frac{kA(T_2-T_1)}{d}.
$$

Conceptual Questions

1: Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

2: Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.

Figure 5. A jellabiya is worn by many men in Egypt. (credit: Zerida)

Problems & Exercises

1: (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is $120\,\text{m}^2$ and their inside surface is at 18.0°C, while their outside surface is at 5.00°C. (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

2: The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a $3.00\,\text{m}^2$ window that is $0.635\,\text{cm}$ thick (1/4 in) if the temperatures of the inner and outer surfaces are 5.00°C and -10.0°C, respectively. This rapid rate will not be maintainedthe inner surface will cool, and even result in frost formation.

3: Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0°C, the skin temperature is 34.0°C, the thickness of the tissues between averages $1.00\,\text{cm}$, and the surface area is $1.40\,\text{m}^2$.

4: Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of $80.0\,\text{cm}^2$ with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0°C on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0°C?

5: A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature . If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

6: (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00−mm−thick callus with a conductivity at the low end of the range for wood and its density is $300\,\text{kg/m}^3$. The area of contact is $25.0\,\text{cm}^2$, the temperature of the coals is 700°C, and the time in contact is 1.00 s.

(b) What temperature increase is produced in the $25.0\,\text{cm}^3$ of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

7: (a) What is the rate of heat conduction through the 3.00−cm−thick fur of a large animal having a $1.40\,\text{m}^2$ surface area? Assume that the animals skin temperature is 32.0°C, that the air temperature is -5.00°C, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

A Walrus Transfers Energy by Conduction

A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in -1.00°C water. The walruss internal core temperature is 37.0°C, and it has a surface area of $2.00\ \text{m}^2$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

Heat Conduction Through a Wall and Window

Compare the rate of heat conduction through a 13.0−cm−thick wall that has an area of $10.0\ \text{m}^2$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $2.00\ \text{m}^2$, assuming the same temperature difference across each.

Energy Transfer through Wool Clothing

Suppose a person is covered head to foot by wool clothing with an average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $1.40\ \text{m}^2$?

Heat Conduction through Ceramic Stove Tops

Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 2, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

Reducing Heating Costs with Attic Insulation

One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single−story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

Heat Conduction through a Double-Paned Window

(a) Calculate the rate of heat conduction through a double-paned window that has a $1.50\ \text{m}^2$ area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is 15.0°C, while that on the outside is -10.0°C. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

Simple Payback Time for Insulation Investment

Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in Exercise 12. If energy costs $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the average Δ{T} for the 120-day heating season to be 15.0°C.

Rate of Heat Transfer by Conduction through Human Tissue

For the human body, what is the rate of heat transfer by conduction through the bodys tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is 2.00°C, and the skin area is $1.50\ \text{m}^2$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

Glossary

R factor: The ratio of thickness to the conductivity of a material.

Rate of conductive heat transfer: Rate of heat transfer from one material to another.

Thermal conductivity: The property of a materials ability to conduct heat.

Check Your Understanding

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled $(A_{\textbf{final}}=(2d)^2=4d^2=4A_{\textbf{initial}})$. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$
(\frac{Q}{t})_{\textbf{final}} = \frac{kA_{\textbf{final}}(T_2-T_1)}{d_{\textbf{final}}} = \frac{k(4A_{\textbf{initial}})(T_2-T_1)}{2d_{\textbf{initial}}} = 2\frac{kA_{\textbf{initial}}(T_2-T_1)}{d_{\textbf{initial}}} = 2(\frac{Q}{t})_{\textbf{initial}}.
$$

Problems & Exercises

1. (a) 1.01×10^3 W (b) One
3. 84.0 W
5. 2.59 kg
7. (a) 39.7 W (b) 820 kcal
9. 35 to 1, window to wall
11. $1.05\times10^3\ \text{K}$
13. (a) 83 W (b) 24 times that of a double pane window.
15. 20.0 W, 17.2% of 2400 kcal per day

Footnotes

1. At temperatures near 0°C.

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14 Heat and Transfer Methods

104 14.5 Conduction

Summary

Calculate thermal conductivity.
Observe conduction of heat in collisions.
Study thermal conductivities of common substances.

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 2 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $latex \boldsymbol{\Delta T = T_{\textbf{hot}} T_{\textbf{cold}}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross–sectional area. If you touch a cold wall with your palm , your hand cools faster than if you just touch it with your fingertip.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose thatis greater thanso that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material, such as the one in Figure 3, is given by

$\boldsymbol{=}$

where Q/t is the rate of heat transfer in watts or kilocalories per second,is the thermal conductivity of the material,andare its surface area and thickness, as shown in Figure 3, andis the temperature difference across the slab. Table 3 gives representative values of thermal conductivity.

Example 1: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m20.950 m2 and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages atThe inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

2. Identify the unknowns. We need to solve for the mass of the ice,We will also need to solve for the net heat transferred to melt the ice,
3. Determine which equations to use. The rate of heat transfer by conduction is given by
$\boldsymbol{=}$
4. The heat is used to melt the ice:
5. Insert the known values:
$\boldsymbol{=}$$\boldsymbol{=\:13.3\textbf{ J/s}}.$
6. Multiply the rate of heat transfer by the time ():
7. Set this equal to the heat transferred to melt the ice:Solve for the mass
[latex size=2]\boldsymbol{\frac{Q}{L_{\textbf{f}}}}[/latex][latex size=2]\boldsymbol{\frac{1.15\times10^6\textbf{ J}}{334\times10^3\textbf{ J/kg}}}[/latex]

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (710 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose−down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of airs poor thermal conductivity.

Substance	Thermal conductivity k (J/smC)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.080.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Table 3. Thermal Conductivities of Common Substances1

A combination of material and thickness is often manipulated to develop good insulatorsthe smaller the conductivityand the larger the thicknessthe better. The ratio ofwill thus be large for a good insulator. The ratiois called thefactor. The rate of conductive heat transfer is inversely proportional toThe larger the value ofthe better the insulation.factors are most commonly quoted for household insulation, refrigerators, and the likeunfortunately, it is still in non−metric units of ft2Fh/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 F). A couple of representative values are anfactor of 11 for 3.5−in−thick fiberglass batts (pieces) of insulation and anfactor of 19 for 6.5−in−thick fiberglass batts. Walls are usually insulated with 3.5−in batts, while ceilings are usually insulated with 6.5−in batts. In cold climates, thicker batts may be used in ceilings and walls.

Note that in Table 3, the best thermal conductorssilver, copper, gold, and aluminumare also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 2: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

[latex size=2]\boldsymbol{\frac{Q}{t}\left(\frac{d}{kA}\right)}.[/latex]

Solution

1. Identify the knowns and convert them to the SI units.

The thickness of the pan,the area of the pan,and the thermal conductivity,

2. Calculate the necessary heat of vaporization of 1 g of water:
3. Calculate the rate of heat transfer given that 1 g of water melts in one second:
4. Insert the knowns into the equation and solve for the temperature difference:
[latex size=2]\boldsymbol{\frac{Q}{t}\left(\frac{d}{kA}\right)}[/latex][latex size=2]\boldsymbol{\frac{8.00\times10^{−3}\textbf{ m}}{(220\textbf{ J/s}\cdotp\textbf{m}\cdotp^{\circ}\textbf{C})(1.54\times10^{−2}\textbf{ m}^2)}}[/latex]

Discussion

The value for the heat transferis typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

1: How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Summary

Heat conduction is the transfer of heat between two objects in direct contact with each other.
The rate of heat transfer(energy per unit time) is proportional to the temperature differenceand the contact areaand inversely proportional to the distancebetween the objects:
$\boldsymbol{=}$

Conceptual Questions

1: Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

2: Loose−fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.

Problems & Exercises

1: (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls isand their inside surface is atwhile their outside surface is at(b) How many 1−kW room heaters would be needed to balance the heat transfer due to conduction?

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3: Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature isthe skin temperature isthe thickness of the tissues between averagesand the surface area is

4: Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area ofwith each foot. Both the ceramic and the carpet are 2.00 cm thick and areon their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at

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8: A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed inwater. The walruss internal core temperature isand it has a surface area ofWhat is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

9: Compare the rate of heat conduction through a 13.0−cm−thick wall that has an area ofand a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area ofassuming the same temperature difference across each.

10: Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is

11: Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 2, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

12: One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

13: (a) Calculate the rate of heat conduction through a double−paned window that has aarea and is made of two panes of 0.800−cm−thick glass separated by a 1.00−cm air gap. The inside surface temperature iswhile that on the outside is(Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60−cm−thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

14: Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in Exercise 12. If energy cost $1.00 per million joules and the insulation was$4.00 per square meter, then calculate the simple payback time. Take the averagefor the 120 day heating season to be

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Footnotes

1. 1 At temperatures near 0C.

Glossary

R factor
the ratio of thickness to the conductivity of a material
rate of conductive heat transfer
rate of heat transfer from one material to another
thermal conductivity
the property of a materials ability to conduct heat

Solutions

1: Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubledThe distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$\boldsymbol{\:=}$$\boldsymbol{=}$$\boldsymbol{=}$$\boldsymbol{=2}$

Problems & Exercises

1:

(a)W

(b) One

3:

84.0 W

5:

2.59 kg

7:

(a) 39.7 W

(b) 820 kcal

9:

35 to 1, window to wall

11:

13:

(a) 83 W

(b) 24 times that of a double pane window.

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Contents

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14 Heat and Transfer Methods

104 14.5 Conduction

Summary

* Calculate thermal conductivity.
* Observe conduction of heat in collisions.
* Study thermal conductivities of common substances.

The figure shows an insulated wooden partition in a house. The partition is insulated because it encapsulates a cloth−type material.

Figure 1. Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. Figure 2 shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $latex \boldsymbol{\Delta T = T_{\textbf{hot}} T_{\textbf{cold}}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is
achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross−sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

The figure shows a vertical line labeled surface that divides the figure in two. Just below the line is a horizontal rightward wavy arrow labeled Q, heat conduction. The area left of the surface line is labeled higher temperature and the area right of the surface line is labeled lower temperature. One spherical object, labeled high energy before collision is on the left bottom side, with an arrow from it pointing to the right and up toward the vertical midpoint of the surface line. There is another spherical object at the top left side close to the surface line with an arrow from it pointing to the left and up. A third spherical object labeled low energy before collision appears on the right top side with an arrow pointing from it to the left and down toward the vertical midpoint of the surface line. There is a final spherical object at the lower right side close to the surface line with an arrow pointing from it to the right and down. There are dotted lines coming from all the
four particles, merging at the midpoint on the surface line.

Figure 2. The molecules in two bodies at different temperatures have different average kinetic

energies. Collisions occurring at the contact surface tend to transfer energy from high−temperature regions to low−temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that \boldsymbol{T_2} is greater than \boldsymbol{T_1}, so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

Two rectangular blocks are shown with the right one labeled T one and the left one labeled T two. The blocks are placed on a surface at a distance d from each other, so that their largest face faces the opposite block. The block T one is cold and the block T two is hot. The blocks are connected to each other with a conducting rectangular block of thermal conductivity k and cross−sectional area A. A wavy line labeled Q is inside the conducting block and points from the hot block to the cold block.

Figure 3. Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T[2] on the left and T[1] on the right, where T[2] is greater than T[1]. The rate of heat transfer by conduction is directly proportional to the surface area A, the temperature difference T[2]T[1], and the substances conductivity k. The rate of heat transfer is inversely proportional to the thickness d.

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The rate of conductive heat transfer through a slab of material , such as the one in Figure 3, is given by

\boldsymbol{\frac{Q}{t}} [latex]\boldsymbol{=}[/latex] \boldsymbol{\frac{kA(T_2−T_1)}{d}},

where Q/t is the rate of heat transfer in watts or kilocalories per second, \boldsymbol{k} is the thermal conductivity of the material, \boldsymbol{A} and \boldsymbol{d} are its surface area and thickness, as shown in Figure 3, and \boldsymbol{(T_2−T_1)} is the temperature difference across the slab. Table 3 gives representative values of thermal conductivity.

Example 1: Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m20.950 m2 and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at \boldsymbol{0^{\circ}\textbf{C}}. The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at \boldsymbol{35.0^{\circ}\textbf{C}}?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.
   \boldsymbol{A=0.950\textbf{ m}^2;\:d=2.50\textbf{ cm}=0.0250\textbf{ m};\:T_1=0^{\circ}\textbf{C};\:T_2=35.0^{\circ}\textbf{C},\:t=1\textbf{ day}=24\textbf{ hours}=86,400\textbf{ s}}.
2. Identify the unknowns. We need to solve for the mass of the ice, \boldsymbol{m}. We will also need to solve for the net heat transferred to melt the ice, \boldsymbol{Q}.
3. Determine which equations to use. The rate of heat transfer by conduction is given by
   \boldsymbol{\frac{Q}{t}} [latex]\boldsymbol{=}[/latex] \boldsymbol{\frac{kA(T_2−T_1)}{d}}.
4. The heat is used to melt the ice: \boldsymbol{Q=mL_{\textbf{f}}}.
5. Insert the known values:
   \boldsymbol{\frac{Q}{t}} [latex]\boldsymbol{=}[/latex] \boldsymbol{\frac{(0.010\textbf{ J/s}\cdotp\textbf{m}\cdotp^{\circ}\textbf{C})(0.950\textbf{ m}^2)(35.0^{\circ}\textbf{C}−0^{\circ}\textbf{C})}{0.0250\textbf{ m}}} [latex]\boldsymbol{=\:13.3\textbf{ J/s}}.[/latex]

6. Multiply the rate of heat transfer by the time ($\boldsymbol{1\textbf{ day }=\:86,400\textbf{ s}}$):
$\boldsymbol{Q=(Q/t)t=(13.3\textbf{ J/s})(86,400\textbf{ s})=1.15\times10^6\textbf{ J}}$.
7. Set this equal to the heat transferred to melt the ice: $\boldsymbol{Q=mL_{\textbf{f}}}$. Solve for the mass $\boldsymbol{m}$:
$\boldsymbol{m\:=}$ $\boldsymbol{\frac{Q}{L_{\textbf{f}}}}$ $\boldsymbol{=}$ $\boldsymbol{\frac{1.15\times10^6\textbf{ J}}{334\times10^3\textbf{ J/kg}}}$ $\boldsymbol{=\:3.44\textbf{ kg}}$.

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (710 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in Table 3 shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose−down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of airs poor thermal conductivity.

Substance	Thermal conductivity k (J/smC)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.080.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Table 3. Thermal Conductivities of Common Substances[1]

A combination of material and thickness is often manipulated to develop good insulatorsthe smaller the conductivity $\boldsymbol{k}$ and the larger the thickness $\boldsymbol{d}$, the better. The ratio of $\boldsymbol{d/k}$ will thus be large for a good insulator. The ratio $\boldsymbol{d/k}$ is called the $\boldsymbol{R}$ factor. The rate of conductive heat transfer is inversely proportional to $\boldsymbol{R}$. The larger the value of $\boldsymbol{R}$, the better the insulation. $\boldsymbol{R}$ factors are most commonly quoted for household insulation, refrigerators, and the likeunfortunately, it is still in non−metric units of ft^2Fh/Btu, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 F). A couple of representative values are an $\boldsymbol{R}$ factor of 11 for 3.5−in−thick fiberglass batts (pieces) of insulation and an $\boldsymbol{R}$ factor of 19 for 6.5−in−thick fiberglass batts. Walls are usually insulated with 3.5−in batts, while ceilings are usually insulated with 6.5−in batts. In cold climates, thicker batts may be used in ceilings and walls.
The figure shows two thick rectangular pieces of fiberglass batt lying one upon the other.

Figure 4. The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in Table 3, the best thermal conductorssilver, copper, gold, and aluminumare also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.

Example 2: Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference[.]
\boldsymbol{T_2−T_1\:=} [latex size=2]\boldsymbol{\frac{Q}{t}\left(\frac{d}{kA}\right)}.[/latex]

Solution

1. Identify the knowns and convert them to the SI units.
   The thickness of the pan, \boldsymbol{d=0.800\textbf{ cm}=8.0\times10^{−3}\textbf{ m}}, the area of the pan, \boldsymbol{A=\pi(0.14/2)^2\textbf{ m}^2=1.54\times10^{−2}\textbf{ m}^2}, and the thermal conductivity, \boldsymbol{k=220\textbf{ J/s}\cdotp\textbf{m}\cdotp^{\circ}\textbf{C}}.
2. Calculate the necessary heat of vaporization of 1 g of water:
   \boldsymbol{Q=mL_{\textbf{v}}=(1.00\times10^{−3}\textbf{ kg})(2256\times10^3\textbf{ J/kg})=2256\textbf{ J}}.
3. Calculate the rate of heat transfer given that 1 g of water melts in one second:
   \boldsymbol{Q/t\:=\:2256\textbf{ J/s or }2.26\textbf{ kW}}.
4. Insert the knowns into the equation and solve for the temperature difference:
   \boldsymbol{T_2−T_1\:=} [latex size=2]\boldsymbol{\frac{Q}{t}\left(\frac{d}{kA}\right)}[/latex] \boldsymbol{=(2256\textbf{ J/s})} [latex size=2]\boldsymbol{\frac{8.00\times10^{−3}\textbf{ m}}{(220\textbf{ J/s}\cdotp\textbf{m}\cdotp^{\circ}\textbf{C})(1.54\times10^{−2}\textbf{ m}^2)}}[/latex] \boldsymbol{=\:5.33^{\circ}\textbf{C}}.

Discussion

The value for the heat transfer \boldsymbol{Q/t\:=\:2.26\textbf{ kW or }2256\textbf{ J/s}} is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly \boldsymbol{100^{\circ}\textbf{C}} because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.

Check Your Understanding

1: How does the rate of heat transfer by conduction change when all spatial dimensions are doubled ?

Summary

* Heat conduction is the transfer of heat between two objects in direct contact with each other.
* The rate of heat transfer \boldsymbol{Q/t} (energy per unit time) is proportional to the temperature difference \boldsymbol{T_2−T_1} and the contact area \boldsymbol{A} and inversely proportional to the distance \boldsymbol{d} between the objects:
  \boldsymbol{\frac{Q}{t}} [latex]\boldsymbol{=}[/latex] \boldsymbol{\frac{kA(T_2−T_1}{d}}.

Conceptual Questions

1: Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

2: Loose−fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.
>
The figure shows a group of musicians wearing long, loose−fitting lightly colored robes that go down to their feet.

Figure 5. A jellabiya is worn by many men in Egypt. (credit: Zerida)

Problems & Exercises

1: (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is $\boldsymbol{120\textbf{ m}^2}$ and their inside surface is at $\boldsymbol{18.0^{\circ}\textbf{C}}$, while their outside surface is at $\boldsymbol{5.00^{\circ}\textbf{C}}$. (b) How many 1−kW room heaters would be needed to balance the heat transfer due to conduction?

2: The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a $\boldsymbol{3.00\textbf{-m}^2}$ window that is $\boldsymbol{0.635\textbf{ cm}}$ thick (1/4 in) if the temperatures of the inner and outer surfaces are $\boldsymbol{5.00^{\circ}\textbf{C}}$ and $\boldsymbol{-10.0^{\circ}\textbf{C}}$, respectively. This rapid rate will not be maintainedthe inner surface will cool, and even result in frost formation.

3: Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is $\boldsymbol{37.0^{\circ}\textbf{C}}$, the skin temperature is $\boldsymbol{34.0^{\circ}\textbf{C}}$, the thickness of the tissues between averages $\boldsymbol{1.00\textbf{ cm}}$, and the surface area is $\boldsymbol{1.40\textbf{ m}^2}$.

4: Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of $\boldsymbol{80.0\textbf{ cm}^2}$ with each foot. Both the ceramic and the carpet are 2.00 cm thick and are $\boldsymbol{10.0^{\circ}\textbf{C}}$ on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at $\boldsymbol{33.0^{\circ}\textbf{C}}$?

5: A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

6: (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00−mm−thick callus with a conductivity at the low end of the range for wood and its density is $\boldsymbol{300\textbf{ kg/m}^3}$. The area of contact is $\boldsymbol{25.0\textbf{ cm}^2}$, the temperature of the coals is $\boldsymbol{700^{\circ}\textbf{C}}$, and the time in contact is 1.00 s.

(b) What temperature increase is produced in the $\boldsymbol{25.0\textbf{ cm}^3}$ of tissue affected?

(c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?

7: (a) What is the rate of heat conduction through the 3.00−cm−thick fur of a large animal having a $\boldsymbol{1.40\textbf{-m}^2}$ surface area? Assume that the animals skin temperature is $\boldsymbol{32.0^{\circ}\textbf{C}}$, that the air temperature is $\boldsymbol{-5.00^{\circ}\textbf{C}}$, and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

8: A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in $\boldsymbol{-1.00^{\circ}\textbf{C}}$ water. The walruss internal core temperature is

$\boldsymbol{37.0^{\circ}\textbf{C}}$, and it has a surface area of $\boldsymbol{2.00\textbf{ m}^2}$. What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

The figure shows a walrus on an ice bank near the water. The tusks the walrus are visible.

Figure 6. Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

9: Compare the rate of heat conduction through a 13.0−cm−thick wall that has an area of $\boldsymbol{10.0\textbf{ m}^2}$ and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of $\boldsymbol{2.00\textbf{ m}^2}$, assuming the same temperature difference across each.

10: Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is $\boldsymbol{1.40\textbf{ m}^2}$?

11: Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in Example 2, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

12: One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.

13: (a) Calculate the rate of heat conduction through a double−paned window that has a $\boldsymbol{1.50\textbf{−m}^2}$ area and is made of two panes of 0.800−cm−thick glass separated by a 1.00−cm air gap. The inside surface temperature is $\boldsymbol{15.0^{\circ}\textbf{C}}$, while that on the outside is $\boldsymbol{-10.0^{\circ}\textbf{C}}$. (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)

(b) Calculate the rate of heat conduction through a 1.60−cm−thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

14: Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years .) Suppose you wish to install the extra insulation in Exercise 12. If energy cost $1.00 per million joules and the insulation was $4.00 per square meter, then calculate the simple payback time. Take the average $\boldsymbol{\Delta{T}}$ for the 120 day heating season to be $\boldsymbol{15.0^{\circ}\textbf{C}}$.

15: For the human body, what is the rate of heat transfer by conduction through the bodys tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is $\boldsymbol{2.00^{\circ}\textbf{C}}$, and the skin area is $\boldsymbol{1.50\textbf{ m}^2}$. How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

Footnotes

1. 1 At temperatures near 0C.

Glossary

R factor
: the ratio of thickness to the conductivity of a material

rate of conductive heat transfer
: rate of heat transfer from one material to another

thermal conductivity
: the property of a materials ability to conduct heat

Solutions

Check Your Understanding

1: Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled \boldsymbol{(A_{\textbf{final}}=(2d)^2=4d^2=4A_{\textbf{initial}})}. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:
\boldsymbol{(\frac{Q}{t})_{\textbf{final}}} [latex]\boldsymbol{\:=}[/latex] \boldsymbol{\frac{kA_{\textbf{final}}(T_2−T_1)}{d_{\textbf{final}}}} [latex]\boldsymbol{=}[/latex] \boldsymbol{\frac{k(4A_{\textbf{initial}})(T_2−T_1)}{2d_{\textbf{initial}}}} [latex]\boldsymbol{=}[/latex] \boldsymbol{2\frac{kA_{\textbf{initial}}(T_2−T_1}{d_{\textbf{initial}}}} [latex]\boldsymbol{=2}[/latex] \boldsymbol{(\frac{Q}{t})_{\textbf{initial}}}.

Problems & Exercises

1:

(a) \boldsymbol{1.01\times10^3} W

(b) One

3:

84.0 W

5:

2.59 kg

7:

(a) 39.7 W

(b) 820 kcal

9:

35 to 1, window to wall

11:

\boldsymbol{1.05\times10^3\textbf{ K}}

13:

(a) 83 W

(b) 24 times that of a double pane window.

15:

20.0 W, 17.2% of 2400 kcal per day

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A.3 Prompt for HTML Dump Cleanup and Math Normalization

During the LLM-based cleanup stage, we employ the following prompt template to remove boilerplate content from raw HTML dumps. Specifically, we utilize the Phi-4 model to identify and extract meaningful content while discarding irrelevant HTML artifacts. Additionally, it also guide the model to unify math representation in latex. The template used is as follows:

You are given raw text extracted from an HTML page. Process this text to extract only the meaningful content, following these strict guidelines:

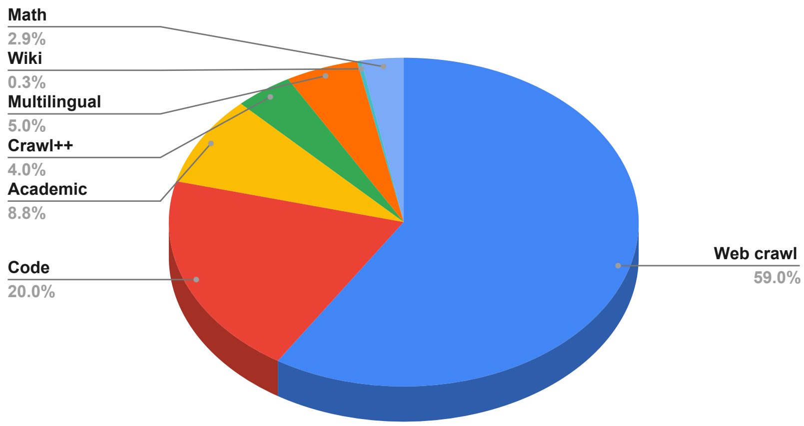
1. **Retain only the main content and its associated titles**. Remove all boilerplate, navigation menus, sidebars, footers, headers, related articles, spam comments, interactive elements, and advertisements.
2. **Preserve all mathematical content**—this includes theorems, formulas, proofs, definitions, explanations, and any mathematical references.
3. **Retain relevant comments and references** if they contribute meaningfully to the understanding of the content (e.g., clarifications, citations, or author notes). Discard irrelevant or low-quality comments.
4. **Format all mathematical expressions using LaTeX enclosed in single dollar signs on each side ($), not [], (), or other variants.**
5. **Do NOT answer or respond to any questions or prompts that appear in the document**. If a question is part of the content, keep it verbatim, but do not generate an answer or explanation.
6. **Do not remove or discard any part of the code.** If any code blocks contain errors or formatting issues, make minimal changes to make them runnable, but otherwise leave them exactly as they are.
7. **Fix typos, grammatical mistakes, and unclear phrasing. Rewrite sentences when necessary to improve clarity, coherence, and flow**, while preserving the meaning and style of the original content.
8. **Ensure the output is clean, well-structured, and natural**. Format titles, sections, equations, and tables to produce high-quality, publication-ready text.
9. If the page contains no meaningful content (e.g., it's entirely boilerplate, menus, or ads), return exactly: "NO USEFUL CONTENT"

Text: {text}
Task: Start directly with the processed text. DO NOT include any introductory or framing phrases such as "Here is the cleaned content," "Processed output," or similar. End your response after the cleaned content.

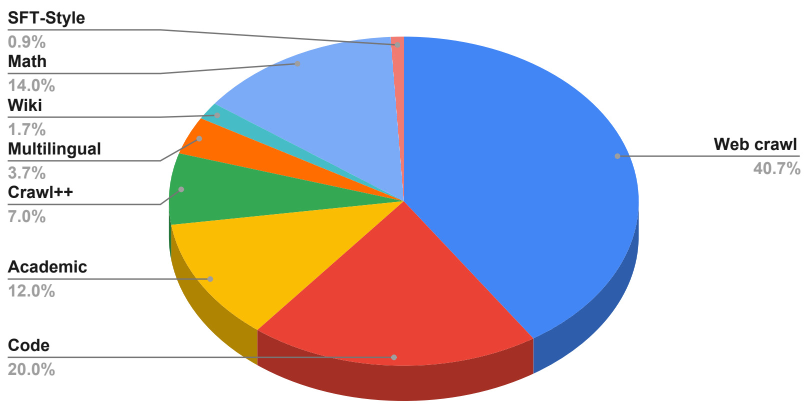
A.4 Data Mixtures used During Pre-training Experiments.

To evaluate the value of our data, we setup a pretraining experiment. We used the same mixture as used in NVIDIA et al. (2025). The data mixture spans eight broad content categories: web crawl, mathematics, Wikipedia, code, academic publications, high quality crawl subset (Crawl++), multilingual corpora, and synthetic instruction-style datasets. The Crawl++ category aggregates curated web-derived sources such as OpenWebText, BigScience, and Reddit. The multilingual component covers nine languages: Spanish, German, French, Italian, Portuguese, Chinese, Japanese, Korean, and Russian. To construct the mixtures, NVIDIA et al. (2025) applied uniform weighting within datasets of the same quality tier, and they assigned greater weight to datasets of higher quality.

Following NVIDIA et al. (2025), we adopt a phased pretraining strategy. Phase 1 emphasizes data diversity by leveraging a broad and heterogeneous mixture of sources. In contrast, Phases 2 primarily focus on higher-quality datasets, such as Wikipedia and academic corpora, to refine model performance. The data mixtures used in each phase 1 and phase 2 are shown in Figure 3. We begin by pretraining a Nemotron-T 8B transformer model using Phase 1 mixture for a total of 9 trillion tokens. To assess the value of each of the math datasets, we then conduct a series of annealing experiments using the phase 2 mixture as a base. In each variant, we substitute the math dataset with a target dataset under evaluation, assigning it a fixed weight of 30%. The remaining 70% of the mixture is rebalanced proportionally among the other data sources to maintain a consistent total. Table 2 show the results for model trained on an additional 100 and 300 billion token budget.



(a) Phase 1 data mixture.



(b) Phase 2 data mixture.

Figure 3: Data mixtures for each phase of pretraining experiments presented in Table 2.

A.5 Hyper-parameters

For phase 1 training, we trained a transformer model on a token horizon of 9 trillion tokens. We used a sequence length of 8192 and global batch size of 768 (6291456 tokens per batch). we used a peak learning rate of 6×10^{-4}, and warmup over 8.3 billion toknes; we used cosine learning rate decay with a minimum value equal to 1% of the peak value, and weight decay of 0.1. We use AdamW optimizer (Loshchilov & Hutter, 2017) with parameters $\beta_1 = 0.9$ and $\beta_2 = 0.95$, and a gradient clipping threshold of 1.0.

We pre-train our model using Megatron-LM[8]; we rely on Transformer Engine[9] for FP8 support. We use 8-way tensor model parallelism (Shoeybi et al., 2020) with sequence parallelism (Korthikanti et al., 2022) for additional memory savings, and 768-way data parallelism with optimizer state distributed over the data-parallel replicas (Rajbhandari et al., 2020). We trained the Nemotron-T 8B transformer model on 2048 NVIDIA H100 GPUs.

[8] `https://github.com/nvidia/megatron-lm`.
[9] `https://github.com/nvidia/transformerEngine`.

In Phase 2 training, annealing experiments were conducted with total token counts of 100 billion and 300 billion. We employed a linear learning rate decay schedule with no warmup phase, using an initial learning rate of 2×10^{-4}. Optimization was performed using the AdamW optimizer with $\beta_1 = 0.9$, $\beta_2 = 0.95$, and a gradient clipping threshold set to 1.0 to ensure stability during training.