An Approximate Mie Scattering Function for Fog and Cloud Rendering

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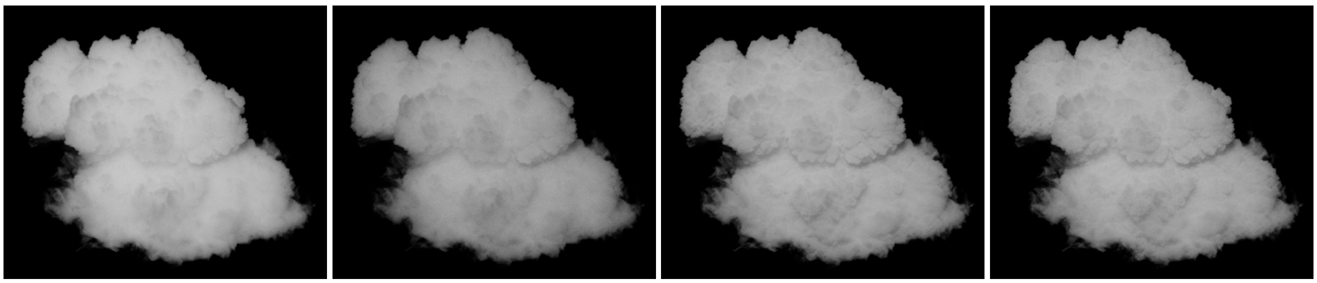

(a) Henyey-Greenstein $g = 0.85$ (b) Double Henyey-Greenstein (c) Tabulated Mie Reference (d) HG-Draine blend (ours)

Figure 1: Path-traced multiple scattering in clouds using a single HG phase function (a) or HG-mixture (b) fails to match a tabulated reference Mie phase function (c). Our model (d) blends a HG forward peak with Draine's phase function to achieve a closer match without requiring large tables for evaluation and sampling. Note the overall brightness and apparent detail in the clouds in (c,d) as compared to the more approximate models (a,b). All renders are 8000 spp and roughly equal time.

ABSTRACT

The Mie phase function describes the complex shapes that arise when light is scattered by water droplets. Inconvenient tables of data are required to include Mie scattering in a path tracer. To avoid this complexity, analytic models such as Cornette-Shanks (CS) or Henyey-Greenstein (HG) mixtures are often used instead, resulting in a lack of accuracy for fog, clouds, skies and tissue. We show that a blend of HG and Draine's phase function can accurately match 95% of the Mie phase function over a wide range of droplet sizes. We provide a practical parameter fit for this mapping and derive analytic CDF inversion of the Draine (and CS) phase function, to produce a parametric approximation with fully analytic evaluation and sampling. In this talk we describe our fitting procedure, sampling derivations, and compare the proposed model to several others.

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1 INTRODUCTION

With the advent of hardware ray tracing, path tracing is increasingly used to render images in games and simulations. The accuracy of path tracing in scenes with atmospheric effects, such as fog and clouds, relies heavily on the phase function's ability to approximate the interaction of light with small particles like water droplets. Unfortunately, the complex shapes (Figure 2) resulting from Mie scattering lack an analytic representation, and their evaluation and sampling necessitate extensive data tables. Consequently, simpler models such as the Henyey-Greenstein (HG) phase function, or their combinations, are often employed, but they are inadequate in certain situations (Figure 1). To increase accuracy and artistic expression, new analytic models are therefore desirable.

To more closely approximate Mie scattering, analytic models such as Cornette-Shanks (CS) [1992] and Draine (D) [2003] have been proposed, but significant errors remain for both, and neither model provides analytic importance sampling, which is essential for real-time applications. In this talk, we show that both CS and Draine phase functions can be sampled analytically, and further, that a linear combination of HG and Draine can accurately fit 95% of the energy (the forward half) of the Mie phase function for water droplets typical of fog and clouds. We performed a parameter fitting for this approximation over a wide range of mean droplet sizes and compared the accuracy to a variety of other approximations. In summary, we present a new two-lobe, parametric, fully-analytic Mie approximation that outperforms previous work.

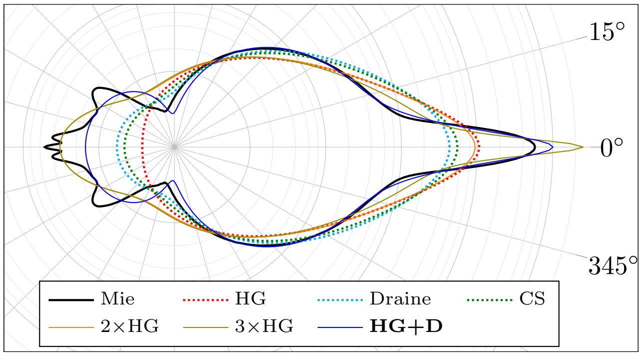

Figure 2: Cross section polar plots of the phase functions.

1.1 Fitting Procedure

We used a publicly available tool [Mie Sim 2020] to generate a tabulated Mie phase function $\phi_{\mathrm{Mie}}(\theta)$ for water droplets in air ($\eta = 1.333$) with log-normal-distributed diameters (standard deviation of 0.25). The results were averaged uniformly over wavelength from 400 nm to 700 nm. Figure 2 shows an example for 5 micron particles (in black).

For the fitting of a target model $\phi(\theta)$, we tried a number of different loss functions and found

$$E_{\mathrm{AS}} = \sum_{\theta} |\cos(\theta)| \sin(\theta) \cdot (\log \phi(\theta) - \log \phi_{\mathrm{Mie}}(\theta))^2 . \quad (1)$$

to produce good results. The sin weighting accounts for integration over the sphere and the cos weighting together with L2 loss in log space helps maintain an accurate mean cosine and increase the influence of backward directions.

2 THE MODEL

We explored a number of different mixtures of HG, Draine, CS, vMF and Gegenbauer lobes and found that a blend of HG (for the peak) and Draine (for the bulk) provided the best fit overall. For a single-wavelength monodisperse simulation, the peak of Mie scattering is asymptotically known to be vMF [Chilton et al. 1969], but we found HG to be a better fit after including fluctuations in particle size and averaging over wavelength.

Draine's phase function is a two-parameter distribution

$$\phi_{\alpha,g}(\theta) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}} \frac{1+\alpha\cos^2\theta}{1+\alpha(1+2g^2)/3} \quad (2)$$

and reduces to HG for $\alpha = 0$, to Rayleigh for $g = 0, \alpha = 1$ and to CS for $\alpha = 1$.

Figure 2 compares a number of isolated models (dashed lines), fitted using Equation (1), as well as several blends of HG lobes. None of these approximations fit the bulk of the distribution as well as our HG+Draine blend, which is remarkably close for the forward half of the phase function (accounting for 95% of the total scattered signal). While our approximation fails to produce the weak back-scattering peaks that lead to fogbow and glory features, these are not observable in most scenes, and our approximation remains the most accurate on average in this region. Performance over mean particle size is shown in Figure 3.

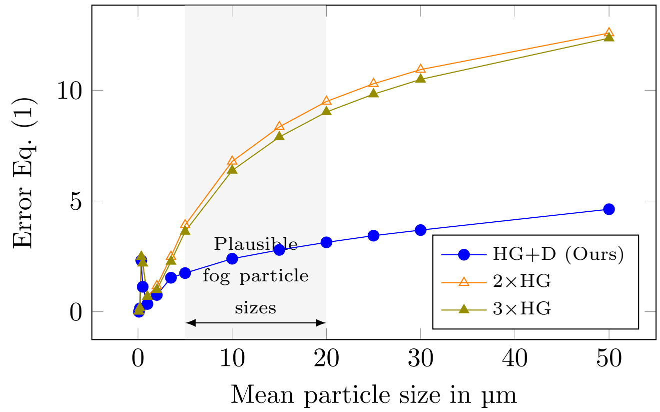

Figure 3: Fitting error of several candidate models.

For sampling cosines u of the Draine phase function (and therefore also CS), we note that the CDF is easily obtained analytically. Upon a substitution of $u \to (1+g^2-v^2)/(2g)$ to remove the square root, solving for random variate ξ is then the solution of a quartic equation (see supplemental for complete details).

2.1 Our Fog Phase Function

Using the combined HG+D phase function, we need four parameters: two anisotropy parameters $g_{\mathrm{HG}}, g_{\mathrm{D}}$, the α parameter of the Draine function and a mixture weight $w_{\mathrm{D}} \in [0,1]$:

$$\phi_{\mathrm{fog}}(\theta) = (1 - w_{\mathrm{D}}) \cdot \phi_{0,g_{\mathrm{HG}}} + w_{\mathrm{D}} \cdot \phi_{\alpha,g_{\mathrm{D}}}. \quad (3)$$

We fitted our model $\phi_{\mathrm{fog}}(\theta)$ over a range of water droplet diameters $5 < d < 50$ in µm and then produced a second fit to map d to the parameters:

$$g_{\mathrm{HG}}(d) = \mathrm{e}^{-\frac{0.0990567}{d-1.67154}}, \quad (4)$$

$$g_{\mathrm{D}}(d) = \mathrm{e}^{-\frac{2.20679}{d+3.91029} - 0.428934}, \quad (5)$$

$$\alpha(d) = \mathrm{e}^{3.62489 - \frac{8.29288}{d+5.52825}}, \quad (6)$$

$$w_{\mathrm{D}}(d) = \mathrm{e}^{-\frac{0.599085}{d-0.641583} - 0.665888} \quad (7)$$

to provide a meaningful and intuitive single parameter for our model. The additional error of this secondary fit is much smaller than the initial fitting error. See the supplemental material for plots of this approximation to reference data, as well as additional mappings for smaller particle diameters.

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