Conditional Resampled Importance Sampling and ReSTIR

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Figure 1: Our new conditional RIS theory enables new types of unbiased subpath reuse by resampling in conditional probability spaces. To show the theory has practical use, we prototype an algorithm resampling multiple ReSTIR-driven path suffixes in a photon map like final gather. While our proof-of-concept is unoptimized, we compare with two state-of-the-art methods without conditional resampling, including a final gather using West et al.’s [2022] marginal multiple importance sampling (MMIS) and full-path resampling using Lin et al.’s [2022] ReSTIR PT sample code. The Tower Bridge [Pohursky 2021] is lit by the Shanghai Bund probe; the camera sees an almost entirely indirectly lit region. While currently more expensive, our subpath resampling gives spatiotemporally stable results without visible correlations. Compared to an MMIS gather, our prototype improves quality given a similar ray budget. (Bottom) Below each image we show \((x,t)\) plots taken from videos (without movement); rows come from sequential video frames, so temporal correlations appear as vertical blobs and spatial correlations show up as horizontal blobs. All techniques are unbiased, converging to reference in time, but results here use only one full path per-pixel for integration.

ABSTRACT

Recent work on generalized resampled importance sampling (GRIS) enables importance-sampled Monte Carlo integration with random variable weights replacing the usual division by probability density. This enables very flexible spatiotemporal sample reuse, even if neighboring samples (e.g., light paths) have intractable probability densities. Unlike typical Monte Carlo integration, which samples according to some PDF, GRIS instead resamples existing samples. But resampling with GRIS assumes samples have tractable marginal contribution weights, which is problematic if reusing, for example, light subpaths from unidirectionally-sampled paths. Reusing such subpaths requires conditioning by (non-reused) segments of the path prefixes.

In this paper, we extend GRIS to conditional probability spaces, showing correctness given certain conditional independence between integration variables and their unbiased contribution weights. We show proper conditioning when using GRIS over randomized conditional domains and how to formulate a joint unbiased contribution weight for unbiased integration.

To show our theory has practical impact, we prototype a modified ReSTIR PT with a final gather pass. This reuses subpaths, postponing reuse at least one bounce along each light path. As in photon mapping, such a final gather reduces blotchy artifacts from sample correlation and reduced correlation improves the behavior of modern denoisers on ReSTIR PT signals.

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2 KEY CHALLENGE IN RIS AND RESTIR

Before exploring prior work and our new theory, let’s review the key difficulty for sample reuse in resampling (e.g., Talbot et al. [2005] and follow ups). In theory, sample reuse is great; the question is “can we?” Considering the high-dimensionality, it is not clear unrelated paths can help integrate over the same domain. Path dimensionality may vary and it is unclear if path measures match; arbitrary sample reuse can feel a bit like asking, “how many grams are in a meter?”

Thus, the work designing ReSTIR [Bitterli et al. 2020; Lin et al. 2022] focused on a few key issues:

- Knowing the sampling domains,
- Matching domains between samples for reuse,
- Shifting samples’ domains to enable more reuse,
- Ensuring we sample the full integration domain, and
- Avoiding double counting parts of the domain.

Discarding sampling domain data is one reason why post-process denoising is fundamentally biased; how can one ensure unbiasedness when averaging final pixel colors that estimate different integrands?

We extend ReSTIR, asking “can we reuse part of a path?” This means readdressing these issues, as subpath reuse makes the domains conditional. For example, if reusing from the 4th path vertex, that subpath was picked relative to the now unused 3rd vertex.

A key difference between our work and prior methods for sampling conditional spaces (e.g., West et al. [2022]) is that we enable a streaming compute model, where reservoirs store individual samples representing aggregations of many (sub-)paths.

3 PREVIOUS WORK AND BACKGROUND

Prior research explores methods for both path reuse and path filtering. Here, we define path reuse as reusing entire samples. Simple methods copy all neighbor path vertices to the current pixel [Bekaert et al. 2002; Ouyang et al. 2021]. Gradient domain rendering added shift mappings, allowing more complex sample reuse between integration domains [Bauszat et al. 2017; Hua et al. 2019]. Similar reuse ideas arise in Metropolis techniques [Veach and Guibas 1997].

Path filtering usually suggests a biased smoothing, typically at path vertices, that connects one path’s prefix to other path suffixes [Binder et al. 2019; Keller et al. 2014; West et al. 2020]. West et al. [2022] noted path filtering can be unbiased, albeit cost prohibitive due to many connections needed in the filtering kernel. We show suffix resampling yields high quality while borrowing just one suffix.
Generally, our theory fits into this reuse and filtering space but allows conditional reuse of subpaths (similar to Tessari et al. [2017] or West et al. [2022]) in the context of resampled importance sampling [Lin et al. 2022], where reused subpaths come from large sample aggregations (reservoirs) from spatiotemporal neighbors.

To demonstrate practical benefits of our theory, we build a proof-of-concept final gather to connect multiple path prefixes to one or more suffixes. Such gathering is not new, per se, being used in photon mapping [Jensen 2001] and path filtering [Binder et al. 2019]. Deng et al. [2021] and West et al. [2022] also showed iterative gathering (or multi-vertex filtering) to be valuable; our theory may enable such ideas in the context of real-time GPU-accelerated ReSTIR variants.

Interestingly, conditional subpath reuse resembles bidirectional path tracing [Lafortune and Willems 1993; Veach and Guibas 1995a] without sampling from lights. Camera paths find key lights; subpaths hitting a light are reused for neighbor pixels as light subpaths. Another view of our prototype is as an unbiased radiance cache, in contrast to prior work that adds bias by estimating density of virtual point lights (traced from the camera) [Segovia et al. 2006] or applying ReSTIR to reuse biased light probes [Majercik et al. 2021].

### 3.2 RIS and ReSTIR

Resampled importance sampling (RIS) [Talbot et al. 2005] and reservoir spatiotemporal importance resampling (ReSTIR) [Bitterli et al. 2020] propose that neighbor sample distributions likely approximate the current integrand better than any analytic distribution. With iterative spatiotemporal bootstrapping, ReSTIR basically draws samples from approximately perfect distributions (see Devroye [1986]).

Similar ideas appear elsewhere, e.g. Metropolis [Veach and Guibas 1997] or simpler path reuse methods [Bekaert et al. 2002], but ReSTIR dramatically accelerates this with streaming computation via weighted reservoir sampling [Chao 1982]. Essentially, ReSTIR allows massive sample amortization, spreading cost over many pixels, without correspondingly higher storage costs.

This builds on the RIS estimator [Talbot et al. 2005], where \( f \) is our integrand (typically the path contribution function):

\[
\langle I \rangle_{\text{RIS}} = \frac{1}{N} \sum_{i=1}^{N} \frac{f(Y_i)}{\hat{p}(Y_i)} \sum_{j=1}^{M} \frac{\hat{p}(X_{ij})}{p(X_{ij})}.
\]

Here, each of the \( N \) samples \( Y_i \) is resampled from \( M \) independent candidates \( X_{ij} \) from some distribution with PDF \( p \). The \( X_{ij} \) are reweighted by some target function \( \hat{p} \), and the \( Y_i \) are drawn from the \( X_{ij} \) proportional to their new weights. These \( N \) samples then estimate \( f \) via Monte Carlo importance sampling.

Standard Monte Carlo estimators sum over samples of the form \( f(Y_i)/p(Y_i) \), but the RIS estimator sums samples \( f(Y_i)W_{Y_i} \), where

\[
W_{Y_i} = \frac{1}{\hat{p}(Y_i)} \sum_{j=1}^{M} \frac{\hat{p}(X_{ij})}{p(X_{ij})}.
\]

Equation 2 includes another Monte Carlo estimator \( \hat{p}(X_{ij})/p(X_{ij}) \). Replacing it by RIS estimator \( \hat{p}(X_{ij})W_{X_{ij}} \) allows chaining RIS estimates, which is the key idea behind ReSTIR [Bitterli et al. 2020].

Sample reuse in RIS that leads to these weights \( W \) is one example of a more general idea, that of unbiased contribution weights introduced by Lin et al. [2022]:

**Definition 3.1.** An unbiased contribution weight (or UCW) for random variable \( X \), is any real-valued random variable \( W_X \) for which

\[
\mathbb{E}[f(X)W_X] = \int_{\text{supp}(X)} f(x) \, dx,
\]

for any integrable function \( f : \Omega \to \mathbb{R} \).

Note that throughout the paper, we denote random variables with capitals.

### 4 CONDITIONAL UCWS

A conditional PDF \( p_{X|Y} \) can be seen as the PDF of \( X \) in a conditional probability space where random variable \( Y \) receives a specified value. Here \( Y \) is constant, so \( f/p_{X|Y} \) estimates the conditional expectation \( \mathbb{E}[f/p_{X|Y}] \). But what if we only have an unbiased estimate of \( 1/p_{X|Y} \), e.g., by conditional RIS (Section 5)?

The estimator \( f/p_{X|Y} \) has the conditional expectation

\[
\mathbb{E} \left[ \frac{f(X)}{p_{X|Y}(X|Y)} \middle| Y \right] = \int_{\text{supp}(X|Y)} f(x) \, dx,
\]

where \( \text{supp}(X|Y) \) contains values \( X \) possible with positive PDF given \( Y \), \( p_{X|Y}(X|Y) > 0 \). Here \( f \) may depend on \( Y \), as \( Y \) is fixed.

This \( \mathbb{E} \left[ \cdot \middle| Y \right] \) can be interpreted as a traditional expectation in the conditional probability space where \( Y \) has fixed value. We observe
that applying UCWs in such a conditional space naturally leads to a definition of conditional unbiased contribution weights.

**Definition 4.1.** A conditional unbiased contribution weight $W_{X|Y}$ for random variable $X$, given $Y$, is any real-valued random variable $W_{X|Y}$ for which

$$E[f(X)W_{X|Y}|Y] = \int_{\text{supp}(X|Y)} f(x) \, dx,$$

(5)

for any integrable function $f : \Omega \rightarrow \mathbb{R}$.

Similar to traditional UCWs, an immediate follow-up is that $W_{X|Y}$ has conditional expectation $E[W_{X|Y} | X, Y] = 1/PX|Y(X|Y)$.

### 4.1 Joint UCWs

Knowing a marginal PDF $p_Y$ and a conditional PDF $p_{X|Y}$ allows unbiased integration with pairs $(X, Y)$: the product $p_Y(y) p_{X|Y}(x|y)$ yields the joint PDF $p_{X,Y}(x,y)$. Does knowing $W_Y$ and $W_{X|Y}$ allow unbiased integration? Let us approach this with an example.

We integrate $f(x,y)$ over the unit square with points $(X_1, X_2)$, with $X_2$ sampled conditionally on $X_1$. Multiplying $f$ by $W_{X_2|X_1}$ estimates the conditional expectation

$$E[f(X_1, X_2)W_{X_2|X_1}|X_1] = \int_{\text{supp}(X_2|X_1)} f(x_1, x_2) \, dx_2,$$

(6)

which integrates over $X_2$ for the fixed $X_1$. The right-hand-side expression only depends on $X_1$, so now multiplying by $W_{X_1}$ yields the integral over the full space:

$$E[ E[f(X_1, X_2)W_{X_2|X_1}|X_1] W_{X_1}] =$$

$$\int_{\text{supp}(X_1)} \int_{\text{supp}(X_2|X_1)} f(x_1, x_2) \, dx_2 \, dx_1.$$

(7)

Is it then true that whenever we have random variables $X_1$ and $X_2$ with UCWs $W_{X_1}$ and $W_{X_2|X_1}$, then $f(X_1, X_2)W_{X_1}W_{X_2|X_1}$ unbiasedly estimates the integral of $f$ over the joint support of $(X_1, X_2)$, making $W_{X_1}W_{X_2|X_1}$ a joint UCW for $(X_1, X_2)$? The answer is no. Without care, this key subtlety can lead to bias (see the supplemental document Section S.1.2 for an example). The expectation of estimator $f(X_1, X_2)W_{X_1}W_{X_2|X_1}$ can be written

$$E \left[ E \left[ f(X_1, X_2)W_{X_1}W_{X_2|X_1} \right] \right],$$

(8)

which differs from Equation 7. To yield the correct expectation, $W_{X_1}$ must move to the outer expectation. This requires $W_{X_1}$ be **conditionally independent** of $f(X_1, X_2)W_{X_2|X_1}$ given $X_1$, i.e., the expressions must not depend on the same random variables, except for $X_1$. This can be achieved by the following rule:

**Theorem 4.1.** If $X_2$ and $W_{X_2|X_1}$ are conditionally independent of $W_{X_1}$, given $X_1$, then

$$W_{X_2, X_1} = W_{X_1}W_{X_2|X_1}$$

(9)

is a joint unbiased contribution weight for $X = (X_1, X_2)$.

If $X_2$ and $W_{X_2|X_1}$ share dependencies with $W_{X_1}$, besides $X_1$, then UCWs $W_{X_1}$ and $W_{X_2|X_1}$ must be conditional on them, i.e., integration must succeed if we treat shared random variables as constant.

### 5 CONDITIONAL RIS AND INTEGRATION

As in Talbot et al. [2005] and Lin et al. [2022], our new conditional unbiased contribution weights can be computed with RIS. In this case, reused samples’ domains often will not cover the full support of integrand $f$. Consider random variable $X_1$ and some dependent random variables arranged into a vector $Z$, and $X_1$’s conditional UCW $W_{X_1|Z}$ (for example, $X_1$ could be a path suffix and $Z$ could be its prefix). By definition, this UCW can integrate any function $f$, potentially dependant on $Z$, in the conditional support $\text{supp}(X_1|Z)$:

$$E \left[ f(X_1)W_{X_1|Z}|Z \right] = \int_{\text{supp}(X_1|Z)} f(x_1) \, dx_1.$$

(10)

But if supp$(X_1|Z)$ does not cover $f$’s support, our estimate is biased. Ensuring unbiasedness requires at least one sample covering otherwise uncovered regions where $f(x) \neq 0$. We add one canonical sample $X_2$ (see Lin et al. [2022]), with UCW $W_{X_2|Z}$ and known to cover $f$’s entire support; the sampling procedure may depend on $Z$.

Combining $f(X_1)W_{X_1|Z}$ and $f(X_2)W_{X_2|Z}$ gives an unbiased estimate if we appropriately pick MIS weights. If samples were in one domain with known PDFs, we could use the balance heuristic,

$$m_1(x|Z) = \frac{p_1(x|Z)}{p_1(x|Z) + p_2(x|Z)},$$

(11)

to give the unbiased estimate

$$m_1(x|Z) \cdot f(X_1)W_{X_1|Z} + m_2(X_2|Z) \cdot f(X_2)W_{X_2|Z}.$$

(12)

The conditional notation $m_i(\cdot|Z)$ has no deeper meaning: we could denote $m_{Z,f}(x)$ as a parametrized function family (e.g., West et al. [2020, 2022]). For future brevity, we occasionally implicitly drop this dependency on $Z$. (In fact, we had already dropped it from $f$).

In practice, samples $X_2$ often arise via resampling, making an unmodified balance heuristic unusable as we use UCWs with unknown PDFs $p_2$. Lin et al. [2022] used ReSTIR target functions $\hat{p}_i$ as PDF proxies for MIS, enabling sample reuse between domains via shift mappings and their Jacobians. Below, we borrow and expand on Lin et al. [2022] by interpreting their expressions with an implicit $Z$ dependency and in a conditional probability space.

#### 5.1 Resampling

Above, we enabled unbiased integration in conditional probability spaces via conditional UCWs. Next, we outline our conditional RIS (CRIS) that generalizes GRIS [Lin et al. 2022].

Take inputs $X_i \in \Omega_i$ with conditional UCWs $W_{X_i|Z}$, where samples $X_i$ and domains $\Omega_i$ may both conditionally depend on $Z$. We must shift $X_i$ into integrand $f$’s domain $\Omega$ with conditioned shift mappings, evaluating $Y_i = T_i(X_i|Z)$, e.g., transforming a neighbor pixel’s suffix path to start from a path prefix in the current pixel (see Section 6).

Then we resample a $Y$ by picking from $Y_i$ in proportion to resampling weights $w_i = m_i(Y_i)\hat{p}(Y_i|\Omega)W_{Y_i|\Omega}T_i^\Omega$, similarly to GRIS. This gives a (now conditional) unbiased contribution weight: $W_{Y|Z} = \frac{1}{\hat{p}(Y)} \sum w_i$, where $w_i$ and $\hat{p}$ may (implicitly) depend on $Z$. Together, $Y$ and $W_{Y|Z}$ integrate properly in $Y$’s conditional support:

$$E \left[ f(Y)W_{Y|Z}|Z \right] = \int_{\text{supp}(Y|Z)} f(y) \, dy.$$
5.2 Integration in the General Case

We now generalize Equation 12, borrowing samples from other domains by shift mapping. Our MIS weights use the different input domains’ target functions \( \hat{p}_i \) (e.g., suffix path radiance) as proxies for unknown conditional PDFs.

We derive the formula by a shortcut: GRIS provides estimate \( f(X)W_{X|Z} \) by selecting \( X \) randomly from the inputs, but we sum probability times contribution over the choices. Similarly to Lin et al.’s [2022] offline estimator, this reduces color noise, but is equivalent for scalar-valued \( f \) when choosing \( \hat{p} = f \).

We assume \( M \) inputs \( X_i \) in different domains, conditioned by \( Z \), and shift them into \( f \)’s domain via \( Y_i = T_i(X_i|Z) \). The generalized conditional UCW estimator, given \( Z \), is then

\[
(I_Z) = \sum_{i=1}^{M} m_i(Y_i|Z) f(Y_i|Z) W_{X_i|Z} |T'(X_i|Z)|,
\]

for MIS weights \( m_i \), integrand \( f \) (e.g., suffix radiance), pre-shift conditional UCW \( W_{X_i|Z} \), and shift Jacobian determinant \( |T'| \). Any MIS weights from Lin et al. [2022] can be used if we interpret the formulas with an implicit conditioning on \( Z \), which we do from now on. We specifically mention the generalized balance heuristic,

\[
m_i(y) = \frac{a_i \hat{p}_{i-j}(y)}{\sum_{j=1}^{M} a_j \hat{p}_{j-i}(y)},
\]

where \( a_i \) are domain weights specifying the relative weight (confidence) given to the samples, and

\[
\hat{p}_{i-j}(y) = \hat{p}_j(T_j^{-1}(y)) |T_j'^{-1}(y)|
\]

reads the (conditional) target function at \( y \)’s corresponding path in pixel \( j \), i.e., \( T_j^{-1}(y) \), as a proxy for its conditional PDF, and the Jacobian determinant transforms this proxy the same way probability densities transform in shift mappings.

This \( (I_Z) \) integrates over the samples’ supports in \( f \)’s domain:

\[
\mathbb{E}[(I_Z)|Z] = \int_{\cup_{i=1}^{M} \text{supp}(Y_i|Z)} f(y) \, dy.
\]

If the union covers the integral, e.g., by including a canonical sample, this estimator is unbiased. We use this estimator to integrate suffix radiance by defining \( Z \) as the supporting prefixes, i.e., unused parts of our reused (sub)paths.

Next, we describe a framework using conditional RIS and ReSTIR in path tracing to reuse suffix paths between pixels and frames.

6 SUFFIX RESTIR

Applying conditional RIS theory to unidirectional path tracing, we build a proof-of-concept prototype\(^1\) that produces well-distributed suffix subpaths and reuses them spatiotemporally. We sample new prefixes at integration, connecting each such prefix to one or more conditionally-sampled suffixes. We update suffixes via ReSTIR, improving their distribution temporally. We summarize our prototype in Algorithm 1 (our supplemental document has more details). Note that reservoir sizes double, versus ReSTIR PT [Lin et al. 2022], as we must store both prefix and suffix data.

Reused suffixes may not entirely cover the suffix domains for new integration prefixes, so we combine with a canonical suffix sample via MIS, to guarantee coverage. This ensures unbiasedness.

Our figures show reconstruction shifts with one-bounce prefixes, but we actually use Lin et al.’s [2022] hybrid shift, postponing reconnection on low-roughness vertices. Prefixes end at the second consecutive high-roughness vertex; the remainder is the suffix.

6.1 Prefix and Suffix Distributions

Reservoirs can reside in many domains, including world space, but we describe the concept in screen space, where each pixel has a reservoir, storing a (yellow) prefix and (red) suffix path. We call the prefix a supporting prefix, as it conditions the suffix random.

\(^1\)Prototype code at: https://github.com/NNLabs/conditional-restir-prototype
variable, determining its support and path space coverage. The supporting prefix is needed for conditional MIS weights, shift mappings and target functions for reuse and integration.

Well-distributed supporting prefixes are vital if suffixes are to capture important light paths. This suggests regularly updating with new independent prefixes. But changing a prefix modifies its suffix’s support and target function, worsening its distribution. We strike a balance by updating prefixes temporally with ReSTIR, without spatial reuse. At each frame (Alg. 1, lines 2-5), we sample a new (blue) supporting prefix candidate for each pixel.

We resample supporting prefixes via ReSTIR with target function

\[ f_p(X^p) \] cancels out during resampling. For MIS weights, we again use the generalized balance heuristic (Equation 15), with all terms conditioned with the supporting prefixes. Dropping \( f_p \) also removes a source of imbalance in the MIS weights, reducing variance.

### 6.3 Integration with Borrowed Suffixes

At integration time, we sample an independent integration prefix \( X^p \) (blue) and search nearby for similar supporting prefixes (circled) in world-space by examining path geometry and borrow their suffixes. We sample a (cyan) canonical suffix to combine its contribution with the borrowed suffixes. Our proof-of-concept uses distance between the end of our integration prefix and supporting prefixes to select suffixes for reuse, as overlapping domains are more likely; exploring other heuristics remains an open question.

To integrate, we substitute suffix contribution \( f_p(X^p, x^s) f_s(x^s) \) into Equation 14 (\( Z \) includes \( X^p \) and all supporting prefixes, which we keep implicit in the following). The estimator \( \langle I \rangle \) for suffix contributions integrates over \( X^p \)’s suffix space \( \Omega^p \):

\[
\mathbb{E} \langle I \rangle | X^p = \int_{\Omega^p} f_p(X^p, x^s) f_s(x^s) \, dx^s.
\]

We then multiply \( \langle I \rangle \) by prefix throughput estimate \( f_p(X^p) W_{X^p} \) to get the joint estimator for the full path integral:

\[
\langle I \rangle = \sum_{i=1}^{M} m_i(Y^s_i) f(X^p, Y^s_i) W_{X^p, Y^s_i},
\]

where \( Y^s_i = T^i(X^p_i) \) is the suffix \( X^p_i \)'s shifted to continue from prefix \( X^p_i \), \( f \) is the full path contribution function, \( m_i \) weights the different suffixes, and \( W_{X^p, Y^s_i} = W_{X^p_i | X^p} W_{Y^s_i | X^p} |T^i(X^p_i)| \) is the joint UCW (Section 4.1) for the full path after shifting the suffix, as the Jacobian transforms \( W_{X^p} \) into \( W_{Y^s} \). One of the \( Y^s_i \) is the canonical suffix that guarantees full coverage of the suffix space.

### 7 FINAL GATHER

In Section 6 we described a way to distribute reusable path suffixes, integrating with short prefixes (Figure 2, right), finding supporting prefixes with similar last vertex geometry, and connecting to their resampled suffixes (red). Shrinking supporting prefix length to one simplifies to ReSTIR PT (middle). Distracting resampling artifacts
can arise if strong outliers get widely reused spatiotemporally. This impoverishes the sample pool, causing correlations. Combining a random prefix with reused subpaths increases path variety. But this adds noise, which we address with a final gather (Figure 2, right).

Photon mapping often uses a similar solution. Querying radiance at primary hits leads to blotchy artifacts. A final gather moves photon queries to the second diffuse hit. Like radiance caching, suffix ReSTIR fills space with cache points (yellow), gathering by tracing integration prefixes (blue). Rather than shooting photons from lights, we sample supporting camera prefixes (yellow), concentrating them in high-throughput areas. Instead of interpolating radiances, we reuse resampled suffixes (red) by shift mapping them to the current integration prefix. Estimates of pixel color are conditioned by the supporting prefixes chosen, and must individually be unbiased. As described in Section 6.3, we pick similar supporting prefixes to maximally match their covered path spaces with our integration prefix. Still, we cannot assume that borrowed suffixes alone cover the integration domain, so by default, each integration prefix must sample a canonical suffix. Suffix paths can be long, so this would be prohibitively expensive. We apply a form of Russian roulette to reap the benefits of ReSTIR suffixes without tracing more canonical suffixes.

Our final gather estimator builds on borrowed suffixes (Section 6.3) with the premise that integration prefixes are relatively cheap. We trace a number $N$ of prefixes and average their estimators (Equation 22), each reusing the suffixes of its closest supporting prefixes. We conceptually include canonical suffixes for all $N$ integration prefixes, specifically including them in our MIS weights—but, by Russian roulette, we replace canonical suffix contributions with zero except for one random prefix, whose contribution is multiplied by $N$. But, the multiplication cancels the mean, resulting in an unweighted contribution. As much of path space is likely covered by borrowed ReSTIR suffixes, this roulette greatly improves rendering efficiency. The canonical suffix is only needed for the non-covered minority.

Assuming symmetrically sampled prefixes $X_i^p$, we can choose prefix $X_i^p$ to include canonical suffix $X_{1i}^s$, with no shift mapping required. The other prefixes $X_j^p$ only have reused suffixes $X_{ij}^s, \ldots, X_{Mi}^s$. This leads to the following final gather estimator:

$$
\langle I_{FG} \rangle = m_1(X_{1i}^s | X_1^p) f(X_1^p, X_{1i}^s) W_{X_1^p, X_{1i}^s} + \frac{1}{N} \sum_{j=2}^{M} \sum_{i=1}^{N} m_1(Y_{ij}^s | X_j^p) f(X_j^p, Y_{ij}^s) W_{X_j^p, Y_{ij}^s}
$$

with definitions analogous to Equation 22. The first line is the contribution from the canonical suffix $X_{1i}^s$ (lines 10-12 in Algorithm 1). The inner sum on the second line does the gather (lines 12-16).

ReSTIR-guided suffixes tend to be high quality, and the integration over the dimension freed by postponing reuse tends to be the variance bottleneck. We seek ReSTIR PT’s low noise without its correlation and present a final gather as a candidate solution, but acknowledge our proof-of-concept approach leaves much yet to do.

## 8 PROOF-OF-CONCEPT EXPERIMENTS

To study our theory’s potential to improve quality without tracing many more independent paths, we built a proof-of-concept algorithm using conditional RIS and ReSTIR, per Sections 6 and 7. We started from Lin et al.’s [2022] ReSTIR PT code and Falcor’s path tracer [Kallweit et al. 2022]. We use RTXDI [NVIDIA 2021] for direct lighting and apply NEE at prefix vertices to account for lighting not covered by suffixes. Results use an NVIDIA RTX 4090 at 1920×1080 with max path length of 12. Our supplemental material contains more implementation and performance details, plus full images and a video studying temporal behavior.

We deem our work mostly theoretical. Experiments focus on uncovering insights on the benefits of conditional RIS theory for subtree reuse. Thus, we have not yet searched for optimally-performing algorithms or implementations. We show some equal-time tests, but our goal is not showing our prototype somehow faster or better, but understanding challenges and identifying promising future work.

All in all, our experiments suggest relatively high potential, with some high-reward research directions such as improving importance sampling of integration prefixes, e.g., with ReSTIR or path guiding.

Driving suffixes with ReSTIR helps greatly. In Figure 3 we compare our suffix reuse (“Ours”) to using independent suffixes (“MMIS”), with a form of marginal MIS [West et al. 2022]. We show an inset from Veach Ajar; all light comes indirectly through a barely-open door. Spatiotemporal suffix reuse greatly improves image quality.

One reused suffix may be enough. Figure 3 also ablates over use of multiple path suffixes during integration. For MMIS, adding more suffixes greatly improves quality; our suffix reservoirs already aggregate multiple suffixes, so adding more provides diminishing returns (similar to Wyman and Panteleev [2021]).

Final gather is important. We postpone reuse by one path vertex (vs. ReSTIR PT) and use Monte Carlo integration on the freed dimensions. This increases noise; our prototype lowers this with a final gather. Figure 4 compares various integration prefix counts. Despite Veach Ajar’s indirect lighting, increasing path prefixes may suffice! In some sense, we convert path tracing into one-bounce integration, as if using an unbiased radiance cache—except we reuse full paths. Integration prefix count gives a natural quality slider for our prototype, but in the future, we hope to approach the quality of many prefixes while using fewer, via better importance sampling.

Russian roulette in the final gather improves efficiency. Figure 4 also compares canonical suffix count. We test one canonical suffix per prefix (right column) versus just one per pixel via Russian roulette (other columns); roulette reduces ray count by 80% with often limited quality impact. Sometimes tracing more canonical suffixes could be worthwhile, especially for offline rendering. See the supplemental document for a study of convergence behavior.
Suffix ReSTIR helps with disocclusion and movement. Figure 5 compares our prototype to ReSTIR PT with camera motion in VEACh AJAR. Screen space ReSTIR suffers “variance lag” around disocclusions as reservoirs get reset where temporal reuse fails. Our prototype does not suffer this lag. Disocclusions still invalidate reservoirs, but when integrating, we find valid reuse candidates from farther away by world-space matching. In some sense, we move ReSTIR to object space but still concentrate computation in visible pixels.

We largely fix ReSTIR spatiotemporal correlations. Figure 1 explores correlations in the Tower Bridge. We render a static video, copying horizontal slices of consecutive frames to rows in the (x, t) plots. This shows temporal and spatial correlations as vertical and horizontal blobs, respectively. ReSTIR PT’s strong spatiotemporal correlations [Sawhney et al. 2022] are not present in our result.

Removing correlations helps with denoising. In Figure 6 we apply a pre-release version of DLSS-BR [NVIDIA 2023] to both ReSTIR PT and our prototype. Despite more raw noise at equal time, the lack of correlations often allows the denoiser to produce better quality with our subpath reuse.

High potential with improved final gather. In Figure 6 we compare our current prototype to ReSTIR PT at equal-time (8 prefixes) and also with higher prefix counts. Figure 7 further studies future potential, assuming a high-quality final gather. ReSTIR suffixes often contain enough information to produce clearer images than ReSTIR PT, showing less coloration, noise, and edge artifacts. This highlights the importance of making final gather cheaper.

9 FUTURE WORK

We believe our new conditional RIS theory offers a promising path to extend real-time rendering to harder light paths. Below we list some interesting future research enabled by our theory:

More efficient final gather. Postponing reuse via a final gather largely eliminates ReSTIR correlations. But we need an efficient final gather for cheap, high-quality prefixes. Better importance sampling may be key: path guiding, more resampling steps, low-discrepancy samples, or better stratification might all offer improvements.

Choosing the right supporting prefixes. For quick prototyping, we used a BVH range search [Evanigou et al. 2021] to find supporting prefixes near our integration prefix. Faster ways to find candidates likely exist; better selection heuristics might also improve quality.

Other theoretical applications. Our prototype shows potential theoretical benefits in the context of a simple final gather. Conditional RIS may also have other uses, e.g., spectral rendering [Weidlich et al. 2022] or volume rendering. Improving specific algorithmic details may yield more practical variants, e.g., better shift maps, variable prefix lengths, multi-vertex reuse, mid-prefix direct lighting, etc.

10 CONCLUSION

We present a new conditional RIS theory, generalizing unbiased contribution weights to allow Monte Carlo integration and resampling even with unknown conditional and joint PDFs. This works for chained RIS, enabling resampling with conditional MIS weights and shift mappings, extending Lin et al. [2022].

We apply our theory to spatiotemporally aggregate suffix paths, driving the conditioning prefixes and reused suffixes by ReSTIR. This allows bidirectional-like path reuse with unidirectional paths, focusing computation in visible regions.

Our proof-of-concept, unbiased final gather combines cheap path prefixes with suffixes reused via conditional RIS. This fills path space with unbiased cached suffixes analogous to photons, but sampled from the camera. Debiasing normally requires many canonical paths; Russian roulette allows skipping all but one. All together, this turns the renderer into a low-dimensional integration over short prefixes. But our prototype remains expensive; we need further algorithmic development for practical applications of conditional resampling.

ACKNOWLEDGMENTS

We want to thank Aaron Lefohn for discussions on and support of this research. Additional thanks to Matt Pharr for discussions and feedback on early ideas and paper drafts. Also, thank you to the anonymous reviewers for pointing out areas for clarification and improvement.

REFERENCES


Figure 3: We compare our ReSTIR-driven suffixes (e–h) to a variant without CRIS, tracing new suffixes every frame (a–d, “MMIS”). All results use four integration prefixes. We vary the suffix count reused for each integration prefix in the final gather. Without ReSTIR-driven suffixes we need many more for good quality (a–d), and the ray count increases quadratically due to the balance heuristic. ReSTIR suffixes are better distributed and give good results even with one suffix (e), avoiding the quadratic cost; Figure 4 shows that increasing prefix count is more cost-effective.

Figure 4: Effect of increasing integration prefixes. Here, each final gather prefix connects to one ReSTIR-driven suffix found via nearest-neighbor search. Image quality improves steadily with increased integration prefix count. We produce a canonical suffix for only one prefix, using Russian roulette, reducing ray count up to 80%. Quality loss from roulette is typically minor (compare the right two columns), except on some glossy surfaces.

Figure 5: Comparing disocclusion artifacts in our prototype and ReSTIR PT using fast camera motion. Because our final gather searches nearby candidates for suffix reuse based on vertices later in the path (not the primary hit point), it avoids ReSTIR PT’s discarded history near screen-space disocclusions. Both methods take one full path sample per pixel per frame, but feature additional rays for reuse.
### Figure 6: Our final gather produces better denoised results than ReSTIR PT, thanks to reduced correlation. Our unoptimized prototype with 8 integration prefixes (74 ms) is about equal-time to ReSTIR PT (76 ms) with increased candidate samples, giving a lower bound on achievable quality at interactive frame rates. Our prototype with 32 prefixes (207 ms) and 128 prefixes (736 ms) achieves superior quality but are more expensive. Future research should increase the achievable prefix count, with potential importance sampling improvements giving a multiplicative effect on the effective count.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Ours with 128 prefixes, denoised</th>
<th>ReSTIR PT</th>
<th>Ours (8 prefixes)</th>
<th>Ours (32 prefixes)</th>
<th>Ours (128 prefixes)</th>
<th>Reference</th>
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</thead>
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<tr>
<td>Tower Bridge</td>
<td>SMAPE (raw) 0.519</td>
<td>0.659</td>
<td>0.512</td>
<td>0.425</td>
<td>SMAPE (denoised) 0.132</td>
<td>0.122</td>
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</table>

### Figure 7: Comparing path tracing, MMIS, ReSTIR PT, and our proof-of-concept with one full path per frame, assuming high-quality final gather. Our prototype and MMIS are here configured roughly equal time, which is much longer than ReSTIR PT’s cost; this figure estimates the potential opened by subpath reuse like a final gather. See Figure 6 for an equal-time comparison. We use 128 prefix samples connecting to one ReSTIR suffix per prefix; MMIS uses 64 prefixes connecting to three suffixes for an improved balance. With subpath reuse, we avoid color shift or correlation artifacts common in ReSTIR PT. All scenes feature a moderately-fast animated camera to prevent over-relying on temporal accumulation. See the supplemental document for the timings.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Ours Prototype Final Gather</th>
<th>Path Tracing</th>
<th>MMIS</th>
<th>ReSTIR PT</th>
<th>Ours</th>
<th>Reference</th>
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<td>0.446</td>
<td>0.354</td>
<td>0.277</td>
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