# Student-T and Beyond: Practical Tools for Multiple-Scattering BSDFs with General NDFs

Eugene d'Eon
edeon@nvidia.com
NVIDIA
New Zealand

## ABSTRACT

We present a practical importance-sampling scheme for the Student-T distribution of visible normals by representing the Student-T NDF as a superposition of Beckmann NDFs. Additionally, we derive a new form of delta tracking to evaluate and sample exact BSDFs with general full-sphere NDFs. These tools permit efficient computation of benchmark BSDF values for the multiple scattering from general (including porous) rough surfaces.

## CCS CONCEPTS

• **Computing methodologies → Reflectance modeling**.

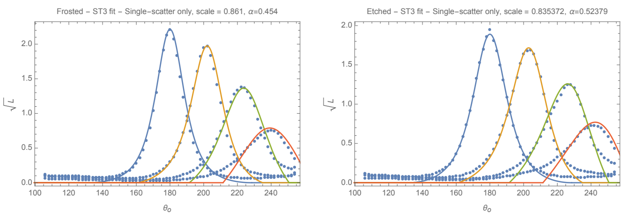
**ACM Reference Format:**
Eugene d'Eon. 2023. Student-T and Beyond: Practical Tools for Multiple-Scattering BSDFs with General NDFs. In *Special Interest Group on Computer Graphics and Interactive Techniques Conference Talks (SIGGRAPH '23 Talks), August 06-10, 2023.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3587421.3595417

## 1 INTRODUCTION

The normal distribution function (NDF) of microfacet theory strongly influences the appearance of rough surfaces. Beckmann and GGX NDFs are most commonly used in rendering, but do not describe all surfaces, and more general distributions are required in some cases. The two-parameter Student-T NDF was introduced to include Beckmann and GGX within a broader family [Ribardière et al. 2017], but has not been compared to measured data. Further, Student-T is limited to describing heightfields, as are other shape-invariant NDFs like the Bessel-K NDF [Bahar and Fitzwater 1983]. To model very rough and porous surfaces, an NDF that extends to the full sphere is required [Dupuy et al. 2016].

Despite the motivation for more general parametric NDFs, in practice only Beckmann and GGX enjoy exact visible-distribution-of-normals (vNDF) sampling procedures that permit low-variance rendering in a path tracer with accurate multiple scattering via random walks. To extend vNDF sampling and random walk multiple scattering to more general NDFs, we present a number of new BSDF tools. Specifically, in this talk we:

(1) Show that Student-T very accurately matches the behaviour of frosted and etched glass (Figure 1)
(2) Derive a vNDF sampling scheme for Student-T
(3) Derive a new form of delta tracking that can produce bias-free reference solutions for surfaces with any bounded NDF
(4) Release a new codebase with validated benchmark implementations of the above methods, behind a new BSDF interface that is easily extendable to new NDFs and permits any BSDF on the microfacets (allowing bi-scale roughness).




(a) Frosted Glass

(b) Etched Glass

**Figure 1: The Student-T NDF produces very close matches to the etched and frosted samples measured previously by [Walter et al. 2007] for $\gamma = 3$, outperforming both Beckmann and GGX fits.**

## 2 THE STUDENT-T NDF

### 2.1 New Physical Motivation

The Student-T NDF was primarily motivated by convenient mathematical properties that are required for efficient use (such as closed-form shadowing/masking factors). To the best of our knowledge, ST BSDFs have not been compared to measured data in order to determine if ST is indeed a good physically-motivated choice for interpolating between Beckmann and GGX. We revisit the original data of transmission through rough glass by [Walter et al. 2007] and show that the ST NDF is indeed a very strong choice, matching the etched and frosted samples very closely. We highlight the accuracy of these two fits in Figure 1 and refer the reader to the supplemental text for additional details.

### 2.2 Sampling the Student-T Distribution of Visible Normals

With notation $u = \cos\theta_m$, the Beckmann NDF of roughness $\alpha$ is

$$D^{\mathrm{B}}(u,\alpha) = \frac{e^{-\frac{\frac{1}{u^2}-1}{\alpha^2}}}{\pi\alpha^2 u^4}\Theta(u), \tag{1}$$

where $\Theta(u)$ is Heaviside's function. The Student-T (ST) NDF with shape parameter $\gamma > 3/2$ is

$$D^{\mathrm{ST}}(u,\alpha,\gamma) = \frac{(\gamma-1)^{\gamma}\alpha^{2\gamma-2}\left(\alpha^2(\gamma-1)+\frac{1}{u^2}-1\right)^{-\gamma}}{\pi u^4}\Theta(u). \tag{2}$$


*SIGGRAPH '23 Talks, August 06-10, 2023, Los Angeles, CA, USA*

ACM ISBN 979-8-4007-0143-6/23/08.
https://doi.org/10.1145/3587421.3595417

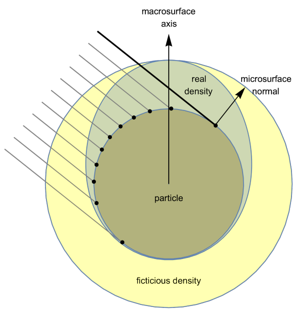



**Figure 2: Illustration of our stochastic null-scattering formulation of non-heightfield Smith scattering.**

The key insight to our method is to note that the ST NDF can be expressed as a gamma-superposition of Beckmann NDFs:

$$D^{\mathrm{ST}}(u, \alpha, \gamma) = \int_0^\infty \frac{e^{-m} m^{\gamma-2}}{\Gamma(\gamma-1)} D^{\mathrm{B}}\left(u, \alpha\sqrt{\frac{\gamma-1}{m}}\right) \mathrm{d}m. \qquad (3)$$

Here $m > 0$ indexes the superposition, and for each $m$ the roughness of the corresponding Beckmann distribution in the mixture is $\alpha\sqrt{(\gamma-1)/m}$.

This superposition can only be used directly for normal incidence ($u = 1$). In the supplemental material, we show how a related distribution $\hat{f}(m, u_{\mathrm{i}})$ can be used to sample superposition roughness $m$ given incident cosine $u_{\mathrm{i}}$ such that sampling Beckmann vNDFs with $m$ results in exact vNDF sampling of ST. New renders of rough gold and diffuse surfaces with multiple-scattering ST BRDFs are included in the supplemental material.

## 3 A NULL-SCATTERING FORMULATION OF ROUGH SURFACE SCATTERING

We also derive a new Monte Carlo procedure for stochastically evaluating and sampling Smith microfacet BRDFs. Our method lifts the heightfield requirement of classical theory (that all microfacet normals lie in the upward hemisphere), allowing porous and purely volumetric materials to be described together with heightfields in a single unified formalism. Our approach takes as input any bounded NDF on the full sphere. Random walks in the volume are then sampled after introducing ficticious/null microflakes over the space of microfacet orientations such that the new NDF is uniform over the sphere, resulting in uniform cross sections and uniform distributions of visible normals. The complex per-NDF derivation of distance-sampling and visible-normal sampling routines of prior work is thereby avoided. The tradeoff is additional cost that is required to perform a rejection procedure analogous to null-collisions for distance sampling in heterogeneous volumes. We show that this approach exactly includes prior height-field Smith models as special cases, but note that the efficiency greatly suffers for low-roughness materials. We anticipate that the approach will be useful for LOD compression of geometric complexity and for modeling very rough / porous / granular / particulate BRDFs (to benchmark faster approximations or train neural models).

In summary our method works by:

- Given a bounded NDF $D(\omega_m) \geq 0$, we select a constant majorant NDF $\bar{D}(\omega_m) \geq D(\omega_m)$ over $S^2$.
- We begin a random walk at the boundary of a half space, along $\omega = -\omega_i$.
- We sample collisions in the medium using the majorant NDF $\bar{D}(\omega_m)$, which reduces to a constant, allowing $\sigma_t = 1$.
- While no actual particles are traced against, the following geometrical procedure illustrates the core sampling idea. For each sampled collision (real or ficticious), we consider a small spherical particle with the pair of NDF values (real and ficticious) mapped to its surface (Figure 2). We then consider a uniform field of rays along direction $\omega$ intersecting this sphere, and sample from this set of rays uniformly at random (2D disk sampling). The intersection of this ray with the sphere produces a microfacet normal $\omega_m$: the normal at the intersected location. This sampled collision is *real* with probability $D(\omega_m)/\bar{D}(\omega_m)$ and otherwise we ignore the collision and sample a new distance.
- For each real collision, the microfacet BSDF is sampled using $\omega$ and $\omega_m$ to determine the next direction.
- Upon exiting the half space, the particle weight (which may deviate from 1 for absorbing microfacets) and direction $\omega$ are returned.

Stochastic evaluation of the BSDF performs the same random walk, but with NEE along $\omega_o$ at each real collision. We numerically estimate the Smith Lambda function for any NDF using a Green's function (the Lambda function for the Dirac delta NDF). See the supplemental pdf and code for additional details.

## 4 CODE AND RESULTS

In the supplemental material we provide a validated reference C++ implementation of the above BSDF tools together with a number of images showing novel BSDFs, such as ST rough metals with multiple scattering, biscale roughness and porous metals with a vMF NDF. The code is implemented behind a new compact interface that allows passing any BSDF and NDF into a common Microsurface class, resulting in a new BSDF that can be recursively applied to another microsurface for biscale roughness.

## ACKNOWLEDGMENTS

We thank Eric Heitz and Andrea Weidlich for helpful input, and Steve Marschner and Bruce Walter for sharing their rough glass measurements.

## REFERENCES

Ezekiel Bahar and Mary Ann Fitzwater. 1983. "Shadowing by non-Gaussian rough surfaces for which decorrelation implies statistical independence." *Radio science*, 18, 04, 566–572. https://doi.org/10.1029/RS018i004p00566.

Jonathan Dupuy, Eric Heitz, and Eugene d'Eon. 2016. "Additional Progress Towards the Unification of Microfacet and Microflake Theories." In: *EGSR (EI&I)*, 55–63. https://doi.org/10.5555/3056507.3056519.

Mickaël Ribardière, Benjamin Bringier, Daniel Meneveaux, and Lionel Simonot. 2017. "STD: Student's t-Distribution of Slopes for Microfacet Based BSDFs." In: *Computer Graphics Forum* 2. Vol. 36. Wiley Online Library, 421–429. https://doi.org/10.1111/cgf.13137.

B. Walter, S.R. Marschner, H. Li, and K.E. Torrance. 2007. "Microfacet models for refraction through rough surfaces." In: *Rendering Techniques (Proc. EG Symposium on Rendering)*. Citeseer, 195–206. http://dx.doi.org/10.2312/EGWR/EGSR07/195-206.